Economics Lecture 2

2016-17

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Course Outline

1 Consumer theory and its applications

1.1 Preferences and utility

1.2 Utility maximization and uncompensated demand

1.3 Expenditure minimization and compensated demand

1.4 Price changes and welfare

1.5 Labour supply, taxes and benefits

1.6 Saving and borrowing

2 Firms, costs and profit maximization

2.1 Firms and costs

2.2 Profit maximization and costs for a price taking firm

3. Industrial organization

3.1 Perfect competition and monopoly

3.2 Oligopoly and games

1.1 Preferences and utility
List of topics

1. Marshall's model of consumer demand

2. Hicks' model of preferences and utility

3. Modern assumptions on preferences and utility:
    1. Completeness
    2. Transitivity
    3. Continuity
    4. Non-satiation
    5. Convexity

Preferences and utility

2. Marshall’s model of consumer demand

Consumer theory and its applications

- Late 19th century orthodoxy Marshall.
  - Principles of Economics, 1890

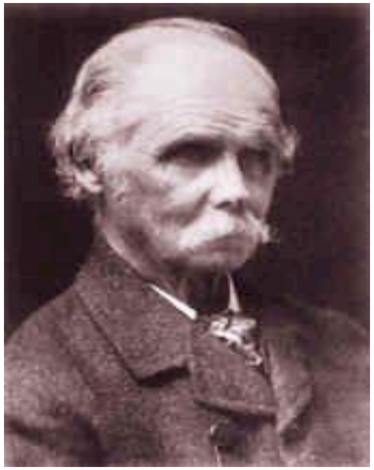
- Mid 20th century orthodoxy Hicks,
  - Value & Capital (1939)

  builds on Edgeworth (1881), Pareto (1909), Slutsky (1915)

- The modern textbook theory .

The purpose of consumer theory

- to understand how household behaviour
  - buying, selling, working, saving and borrowing
- is affected by changes in
  - prices, income, wages, interest rates, tax rates …..
- to provide a way of
  - showing that there are situations in which everyone gains from trade
  - measuring the effect on household welfare of changes in prices, income etc..



Alfred Marshall (1842-1924)

- Cambridge
- Principles of Economics (1890)
- supply and demand curves
- elasticity
- consumer surplus

Marshall's model of consumer demand

- roots in 19th century utilitarian philosophy
- achievements
  - the demand curve
  - consumer surplus
- gaps
  - lots of words, no modelling of
    the effects of income & prices of other goods

Marshall’s theory of consumer demand with integer quantities, 1, 2, 3…

Marshall's assumptions on consumer behaviour

- The consumer chooses quantity x to maximize $V(x) - px$.
- The price p does not depend on x
- $V(x)$ is measured in units of money, €, $V(0) = 0$.
- $V(x)$ is the most a consumer would be willing to pay for x units if the alternative were 0 units.
- Marshall calls $V(x)$ "utility"
- I call $V(x)$ value to avoid confusion with the modern interpretation of utility.

Marshall's assumptions on consumer behaviour

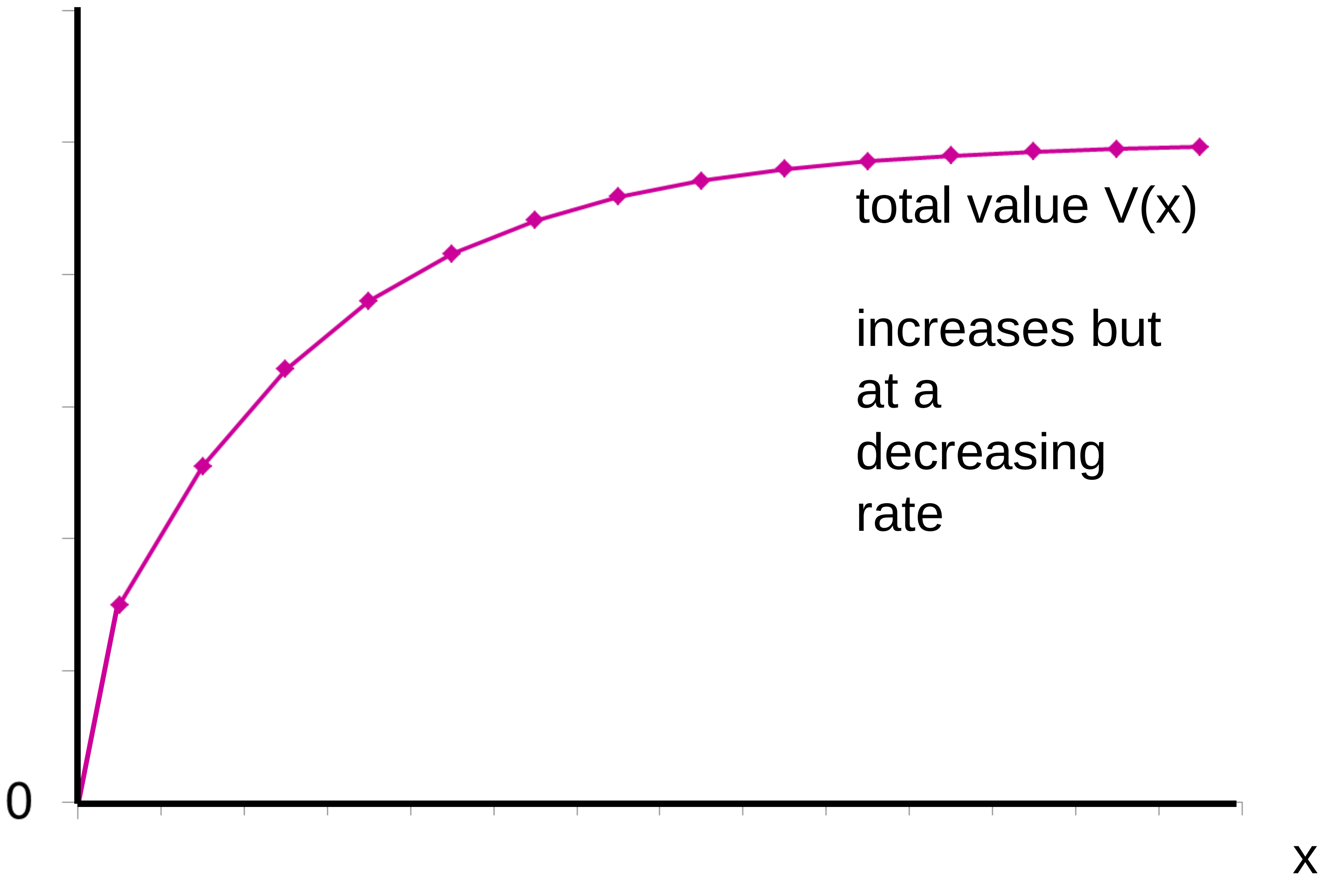
- Marginal value $MV(x) = V(x) - V(x-1)$.

  maximum willing to pay for one more unit.

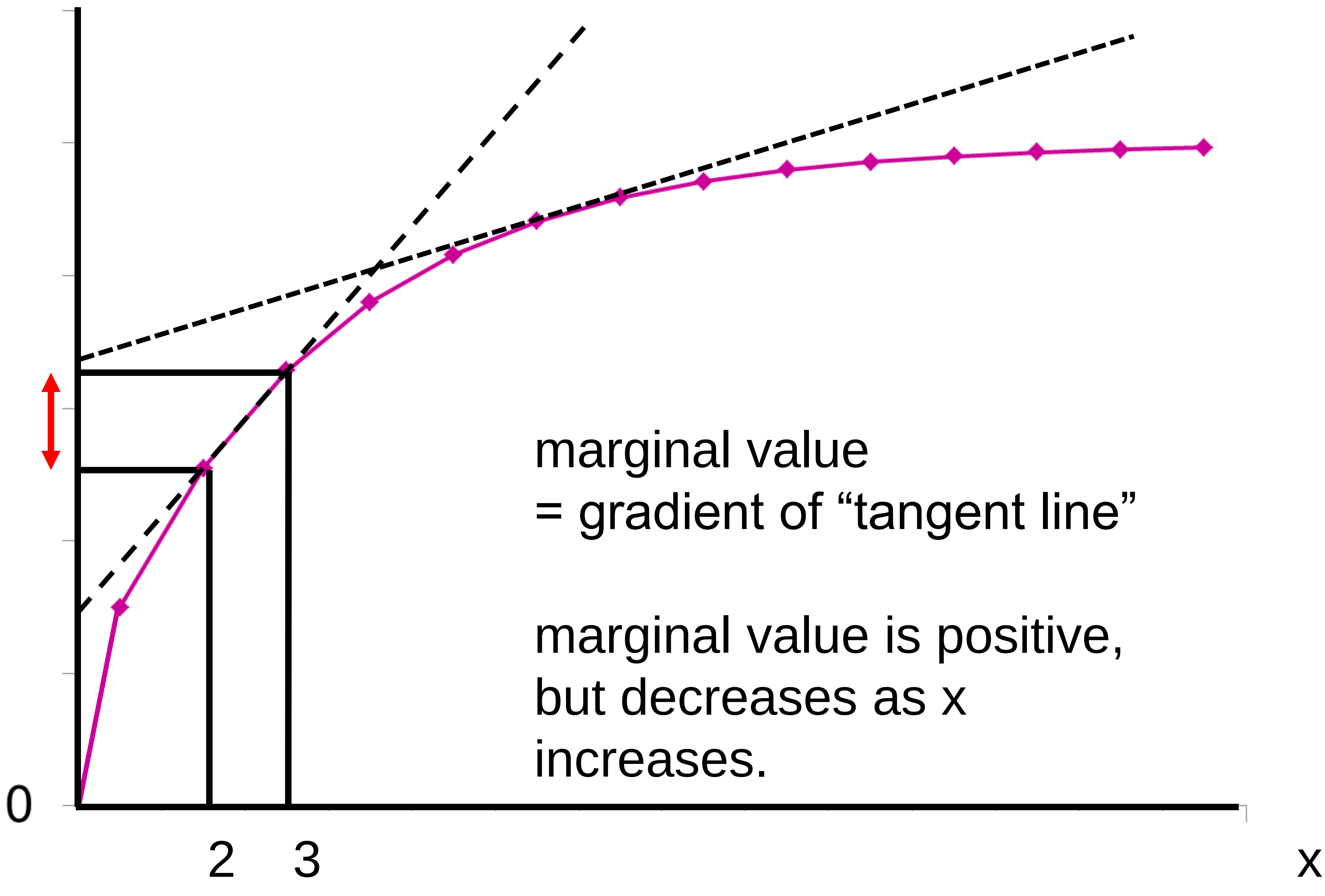
- Marginal value is positive.

- Marginal value decreases as x increases.

  Marshall notes marginal utility may increase with consumption for some goods – e.g. for good music.


total value V(x)
increases but at a decreasing rate
0
x

MV(3) = V(3) – V(2) marginal value

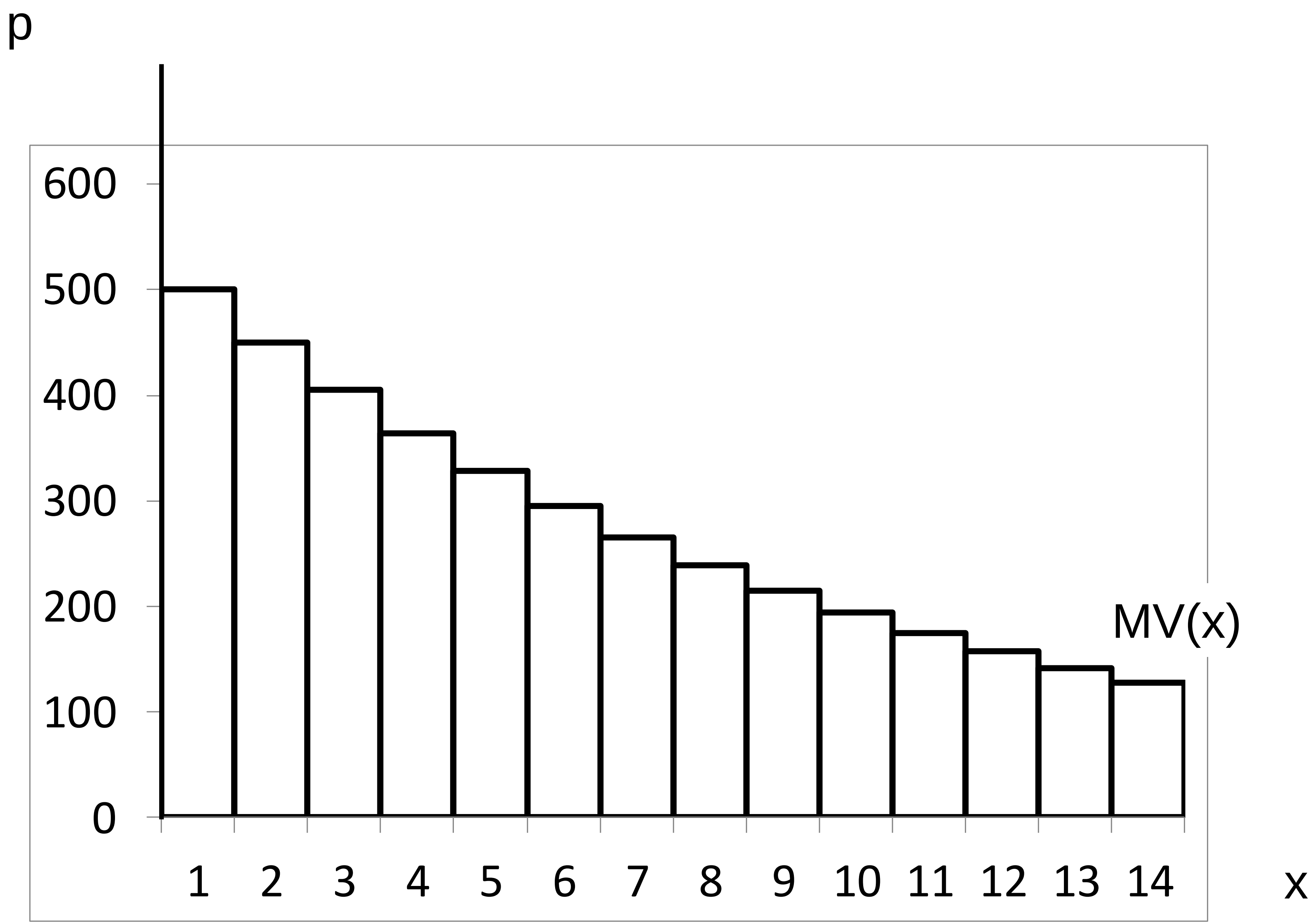


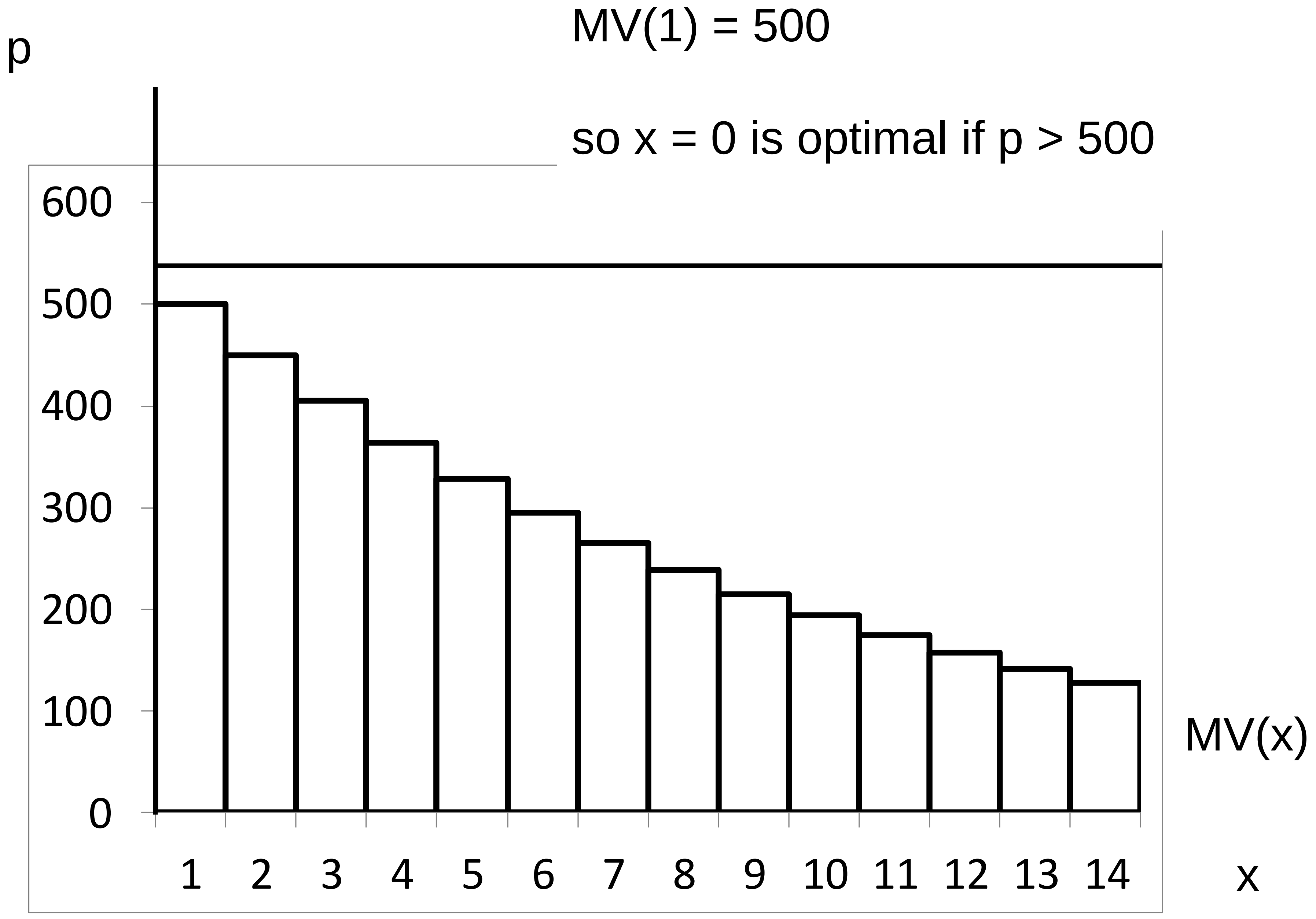
Marginal conditions with integers

- p = price of x
- Objective is to max V(x) – px
- MV(x) = V(x) – V(x - 1) > 0 but decreases as x increases.
- If MV(x) > p increasing from x -1 to x increases V(x) – px
- If MV(x + 1) < p increasing from x to x + 1 decreases V(x) – px
- If MV(x) is decreasing then x is optimal if

$$MV(x + 1) \le p \le MV(x)$$

- If MV(1) < p then x = 0 is optimal.


p
600
500
400
300
200
100
0
MV(x)
1
2
3
4
5
6
7
8
9
10
11
12
13
14
x

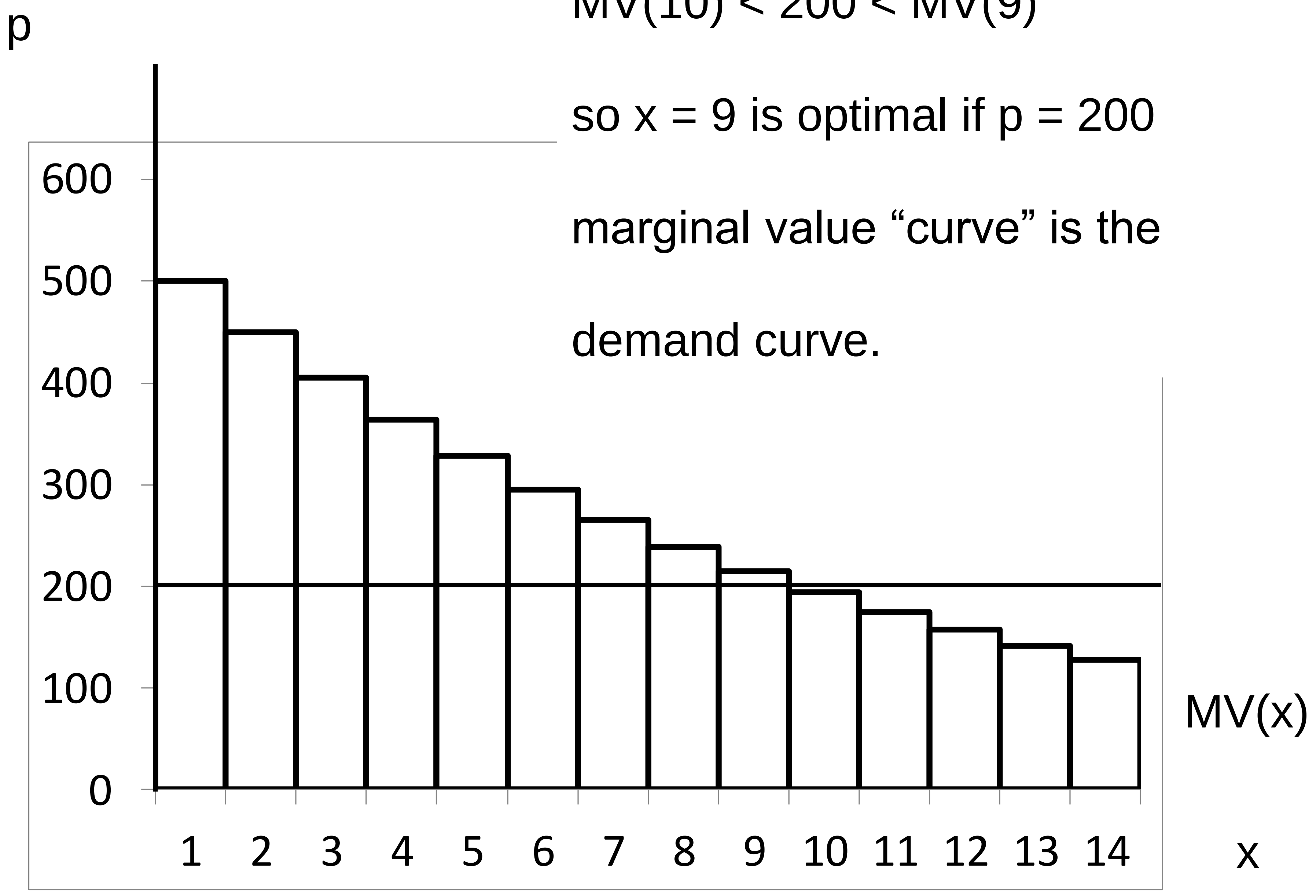

MV(1) = 500
so x = 0 is optimal if p > 500
p
600
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
x
MV(x)

MV(10) < 200 < MV(9)

so x = 9 is optimal if p = 200

marginal value “curve” is the

demand curve.



Marginal value MV and total value V

$V(0)$	=	0
$MV(1)$	=	$V(1) - V(0) = V(1)$
$MV(2)$	=	$V(2) – V(1)$
$MV(3)$	=	$V(3) – V(2)$

Add these equations

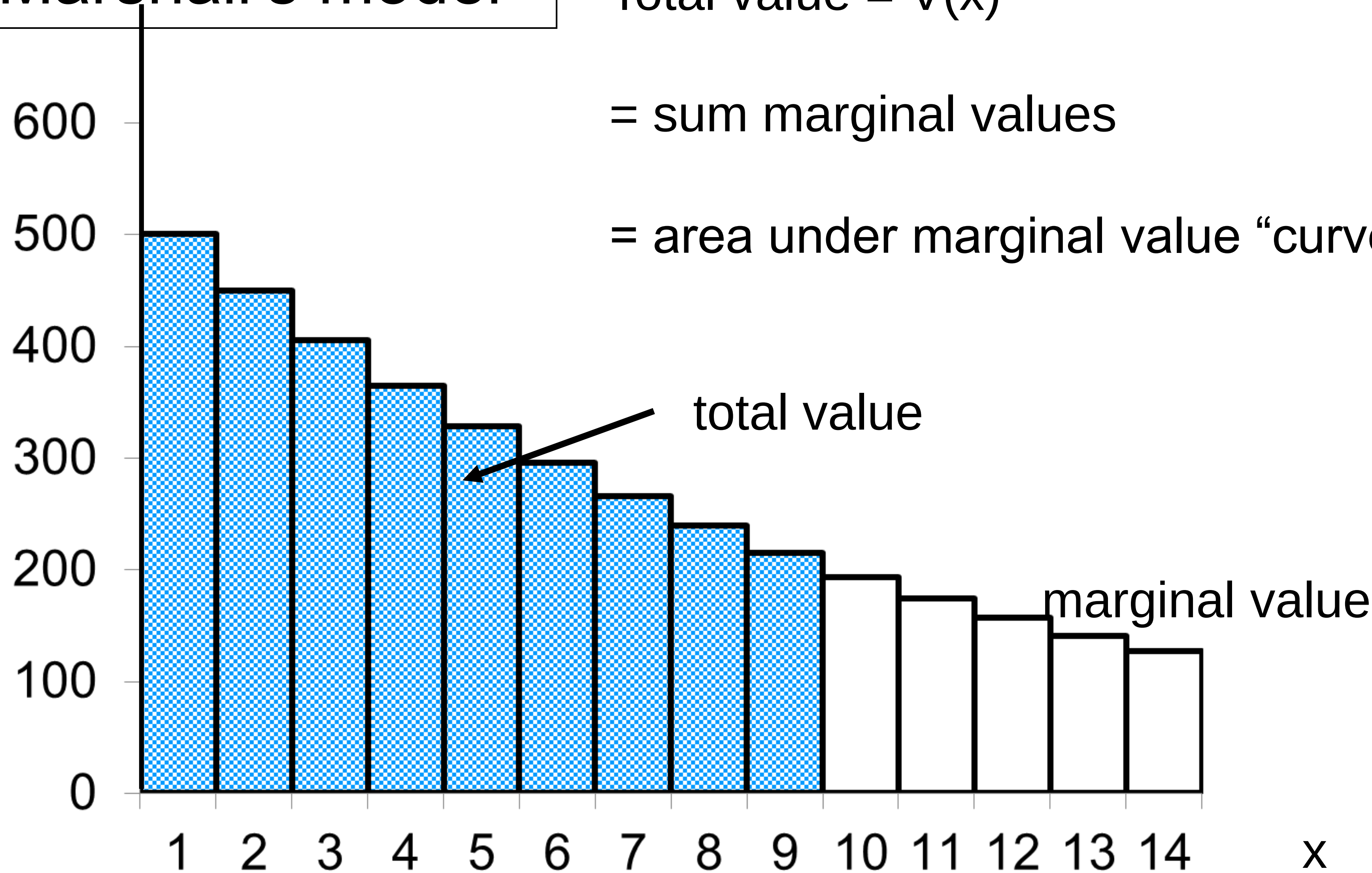
$$MV(1) + MV(2) + MV(3) = V(3)$$

Consumer surplus in Marshall's model

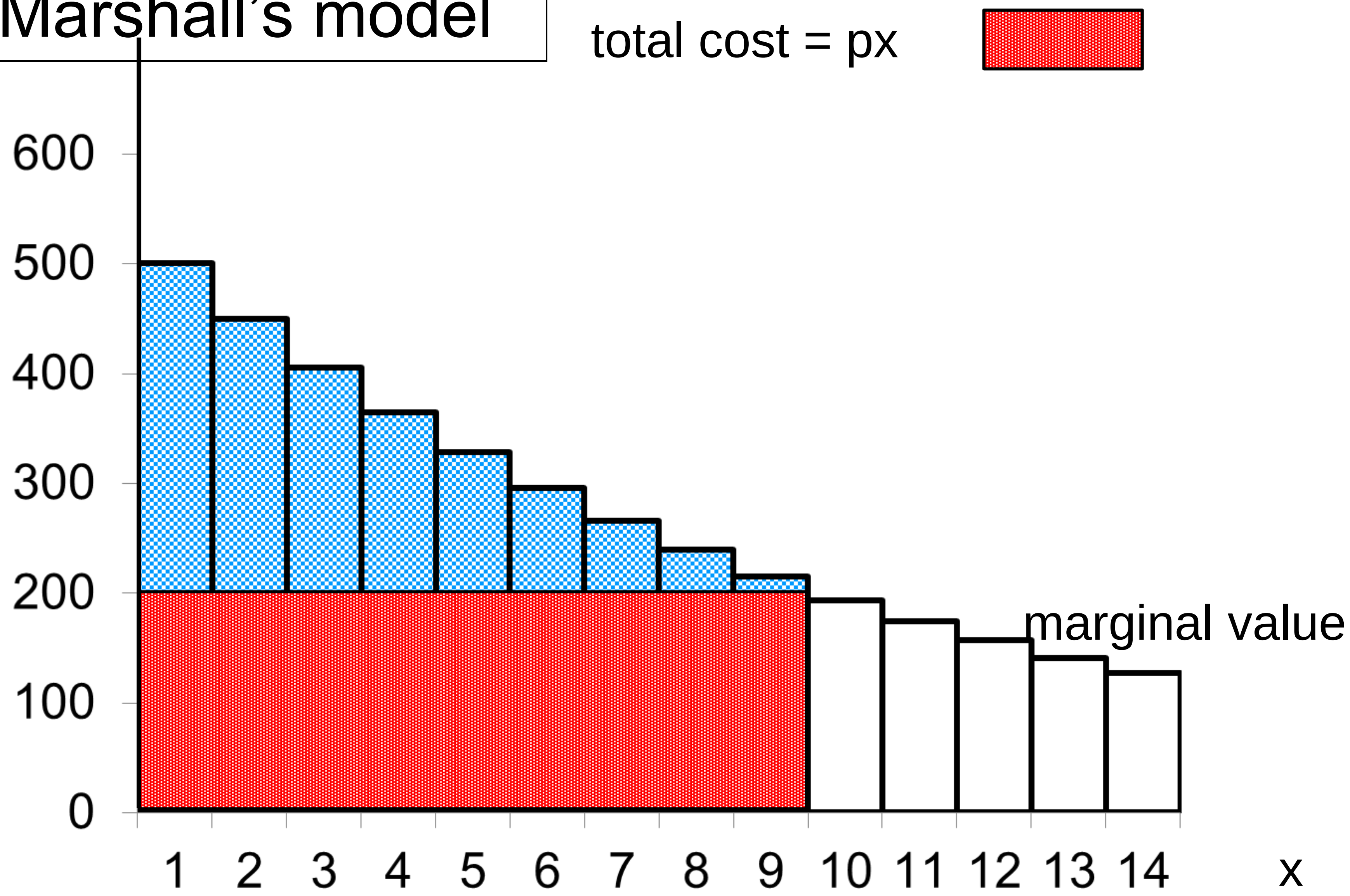
Total value = V(x)

= sum marginal values

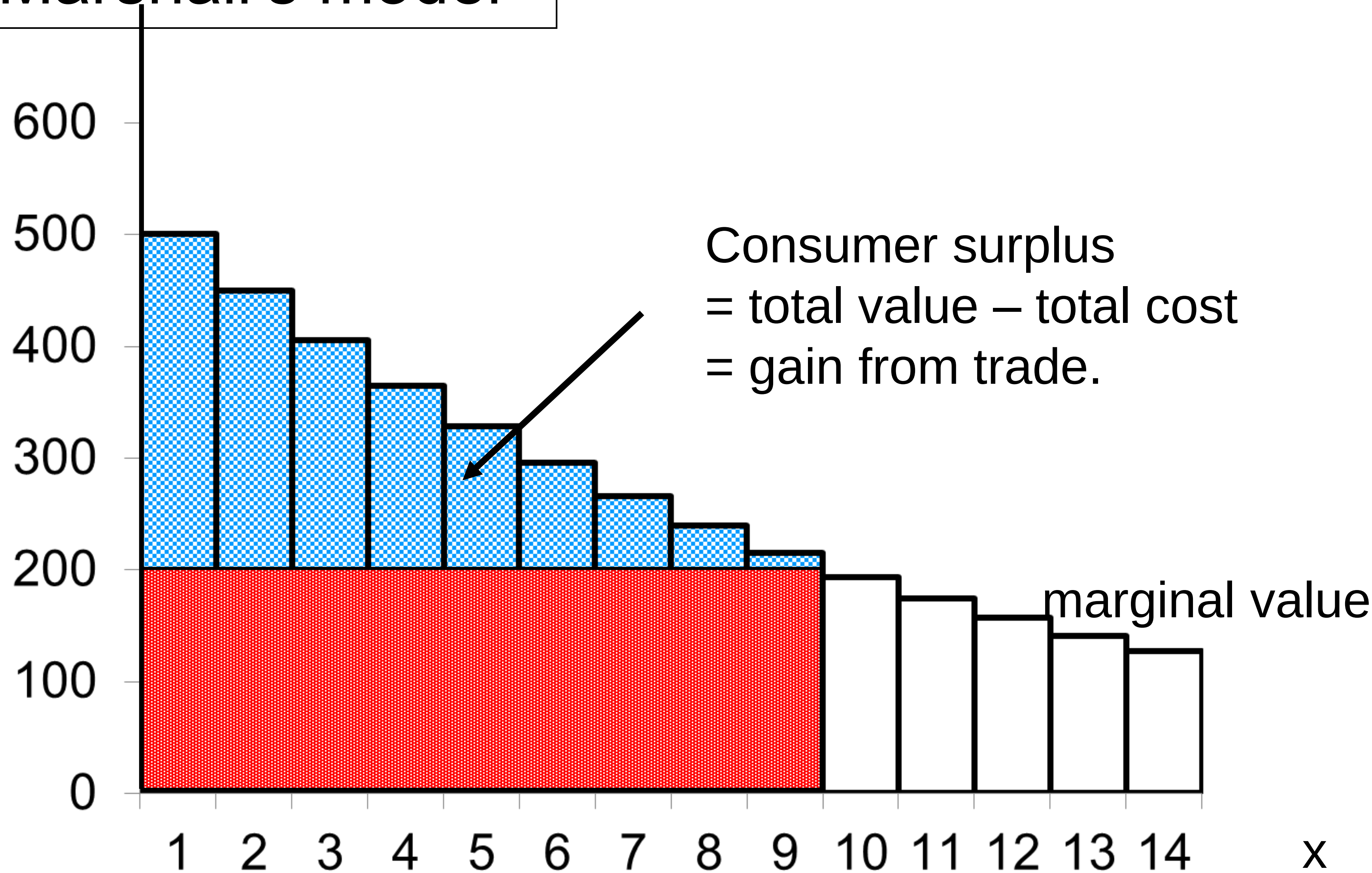
= area under marginal value "curve"



Consumer surplus in Marshall's model



Consumer surplus in Marshall's model



Marshall's theory with calculus

Move to calculus

- Done by working with smaller and smaller units of output.

- Comes from taking limits as the unit size tends to 0.

- Marginal value = gradient of tangent to total curve

  - Differentiate total value

- Total value = area under marginal value curve.

  - Integrate marginal value.

Marshall's assumptions on consumer behaviour in calculus

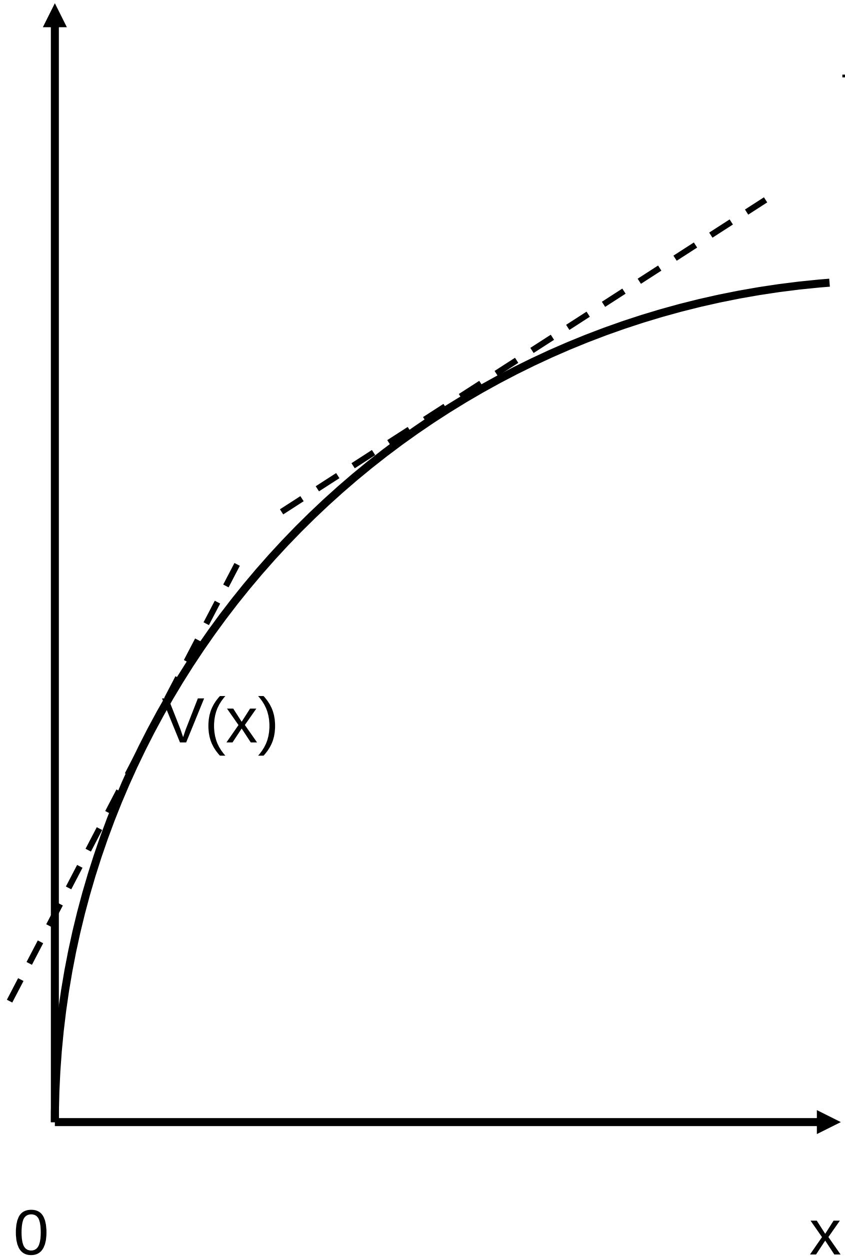
- Total value is an increasing function of x so

  marginal value $\frac{dV}{dx} > 0$

- Marginal value $\frac{dV}{dx}$ decreasing

  $$\frac{d^2V}{dx^2} < 0$$

- Consumers maximize total value – total cost = V(x) - px

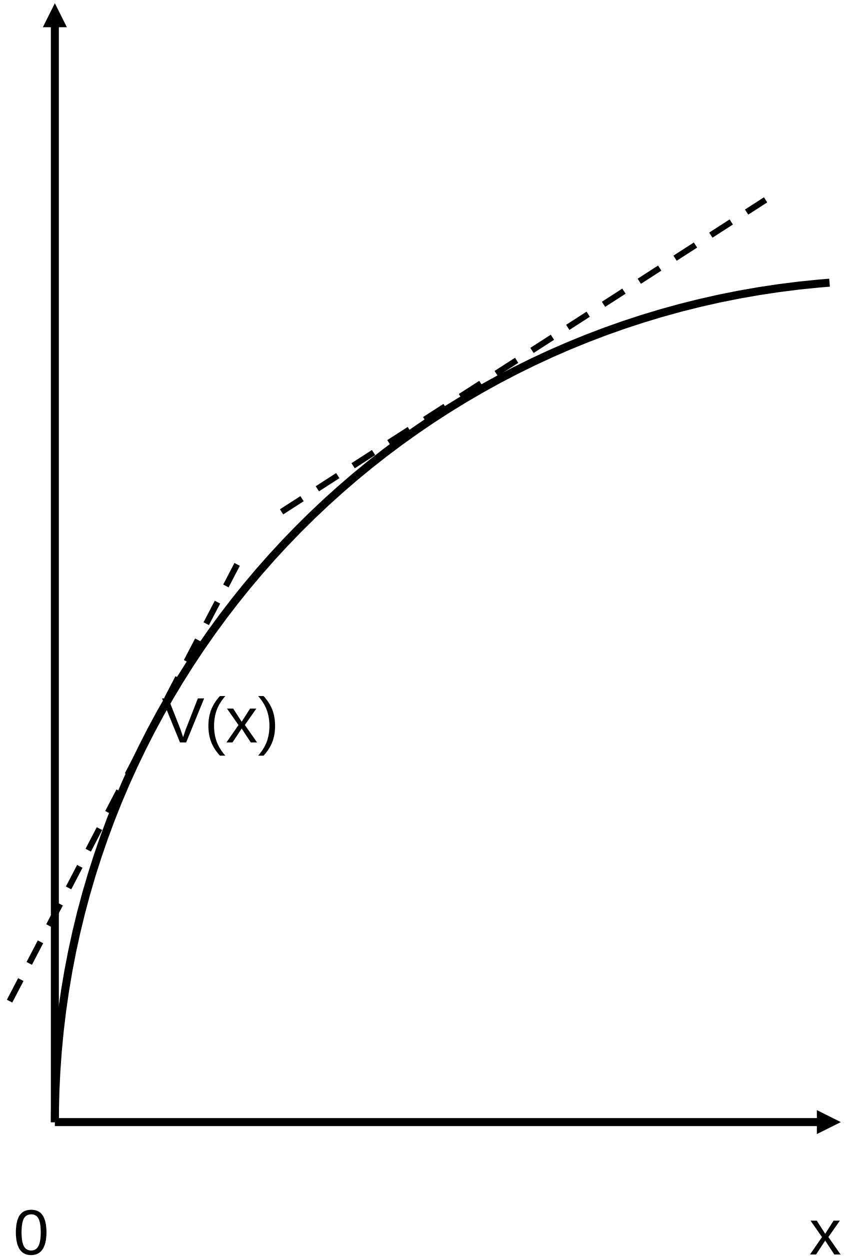


$$\frac{dV}{dx} = \text{marginal value} > 0$$

tangent slopes upwards

total value V(x) is

increasing

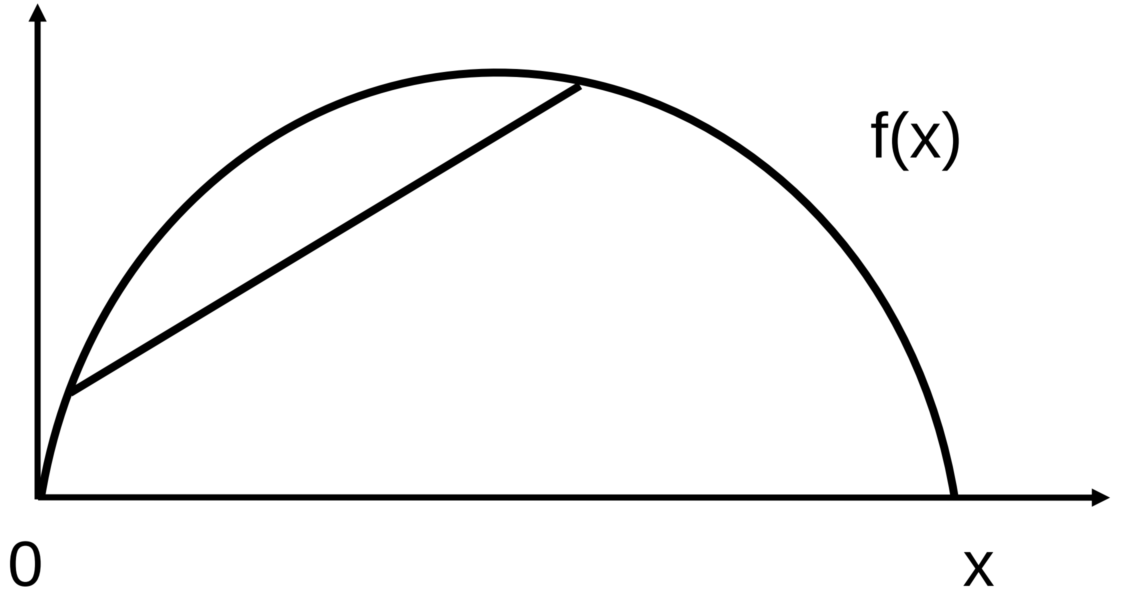


$$\frac{d^2V}{dx^2} < 0$$

$$\frac{dV}{dx}$$ = marginal value is decreasing

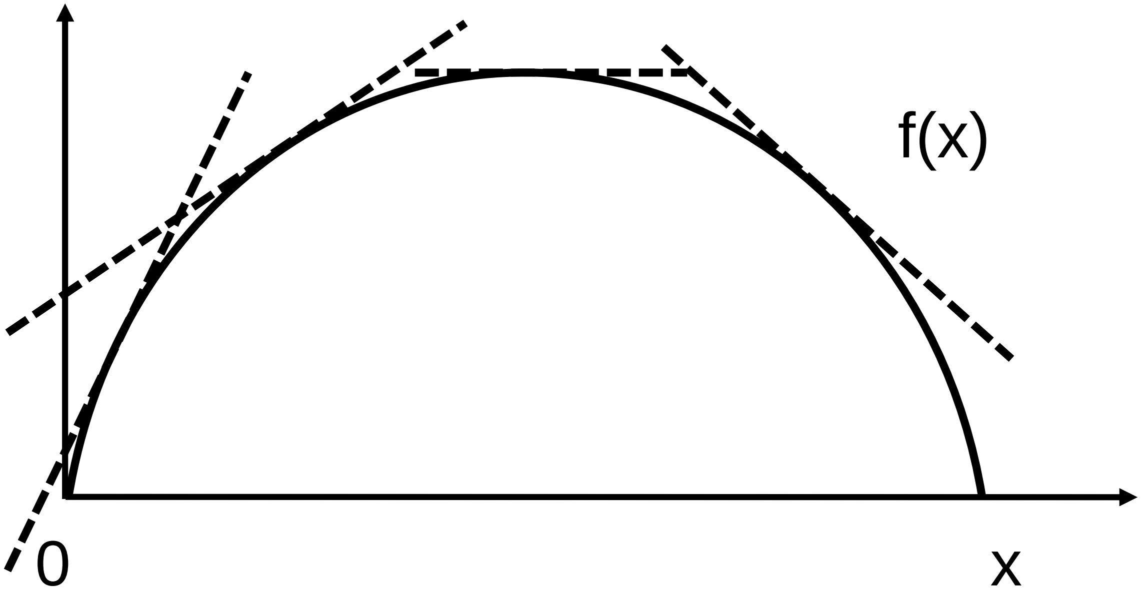
V(x) is concave

Facts about concave functions



A function is concave if any line joining two points on its graph lies entirely on or below the graph.

Facts about concave functions

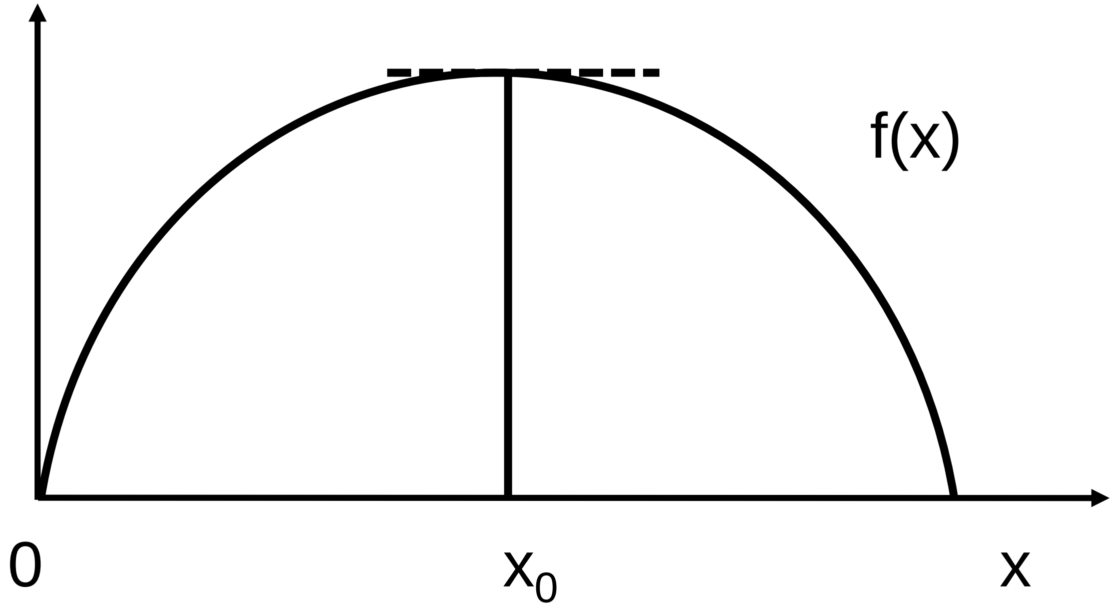


For functions with first and second derivatives

Concave functions are functions with decreasing first derivatives

Concave functions are functions with negative second derivatives.

Facts about concave functions

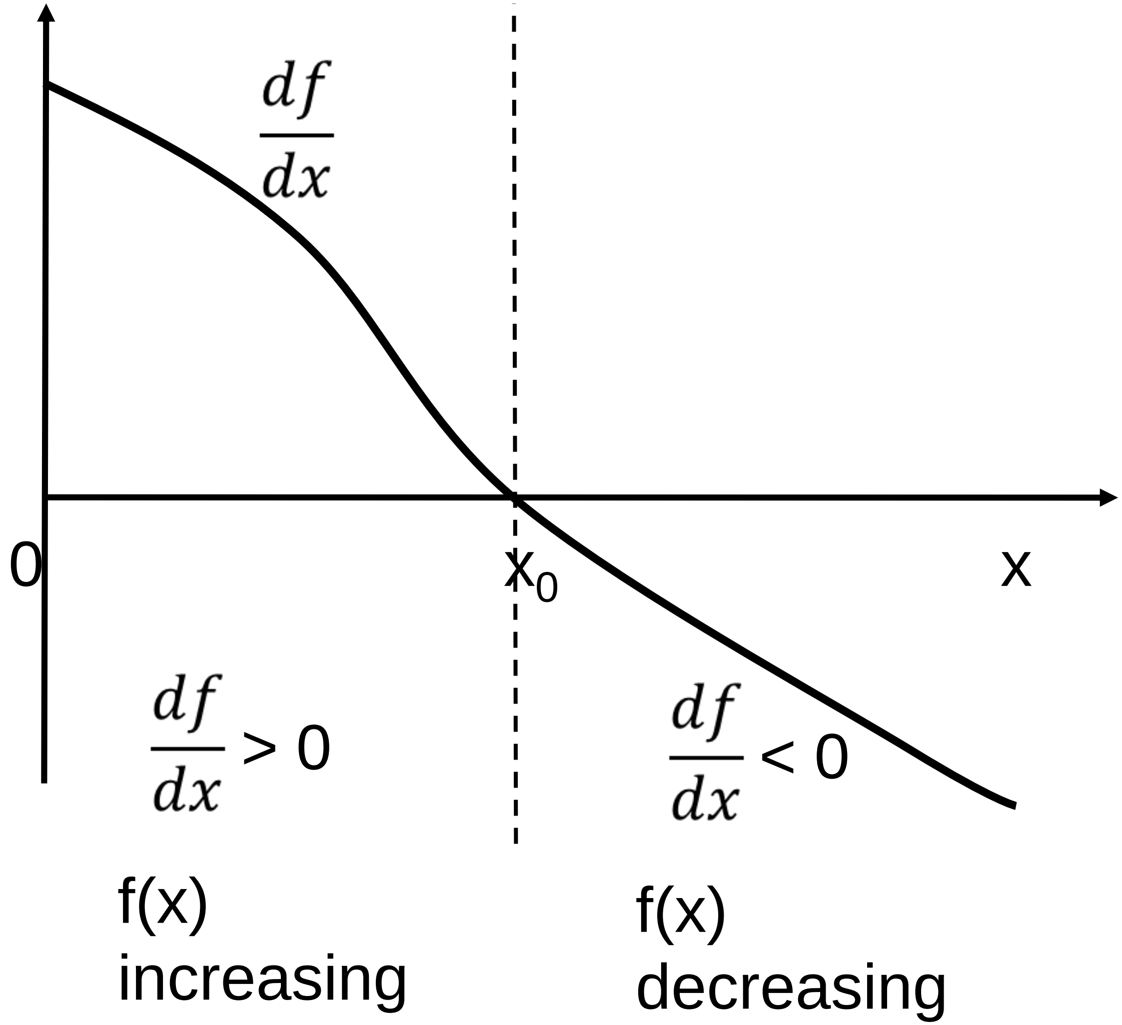


If a function is concave and its derivative is 0 at x_0 then x_0 maximizes the function.

Next slides shows why this is implied by

$\frac{df}{dx}$ decreasing.

Facts about concave functions



At x_0 $\frac{df}{dx} = 0$ thus x_0 maximizes f(x).

Marshall's theory with calculus and concavity

Total value is a concave function of x implies that

total value – total cost = V(x) – px is concave.

Why?

As V(x) is concave the second derivative $\frac{d^2V}{dx^2} < 0$

The second derivative of V(x) – px is also $\frac{d^2V}{dx^2} < 0$

so V(x) – px is concave.

Marshall's theory with calculus and concavity

As V(x) – px is if $x_0 > 0$ concave and the first order condition

$$\frac{dV(x_0)}{dx} = p$$

marginal value = price

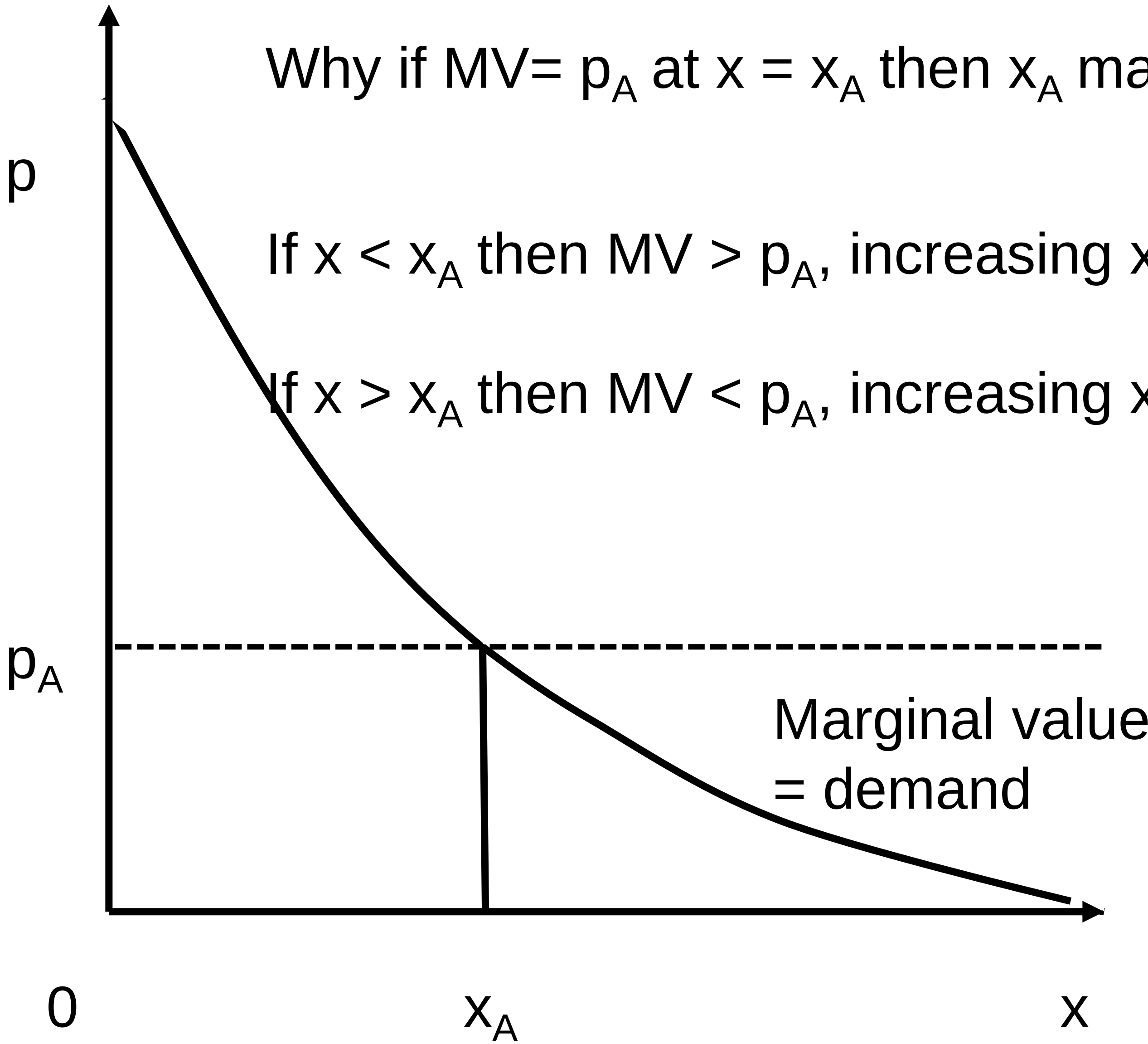
is satisfied then x_0 maximizes total value – total cost

Marshall's Theory with Calculus

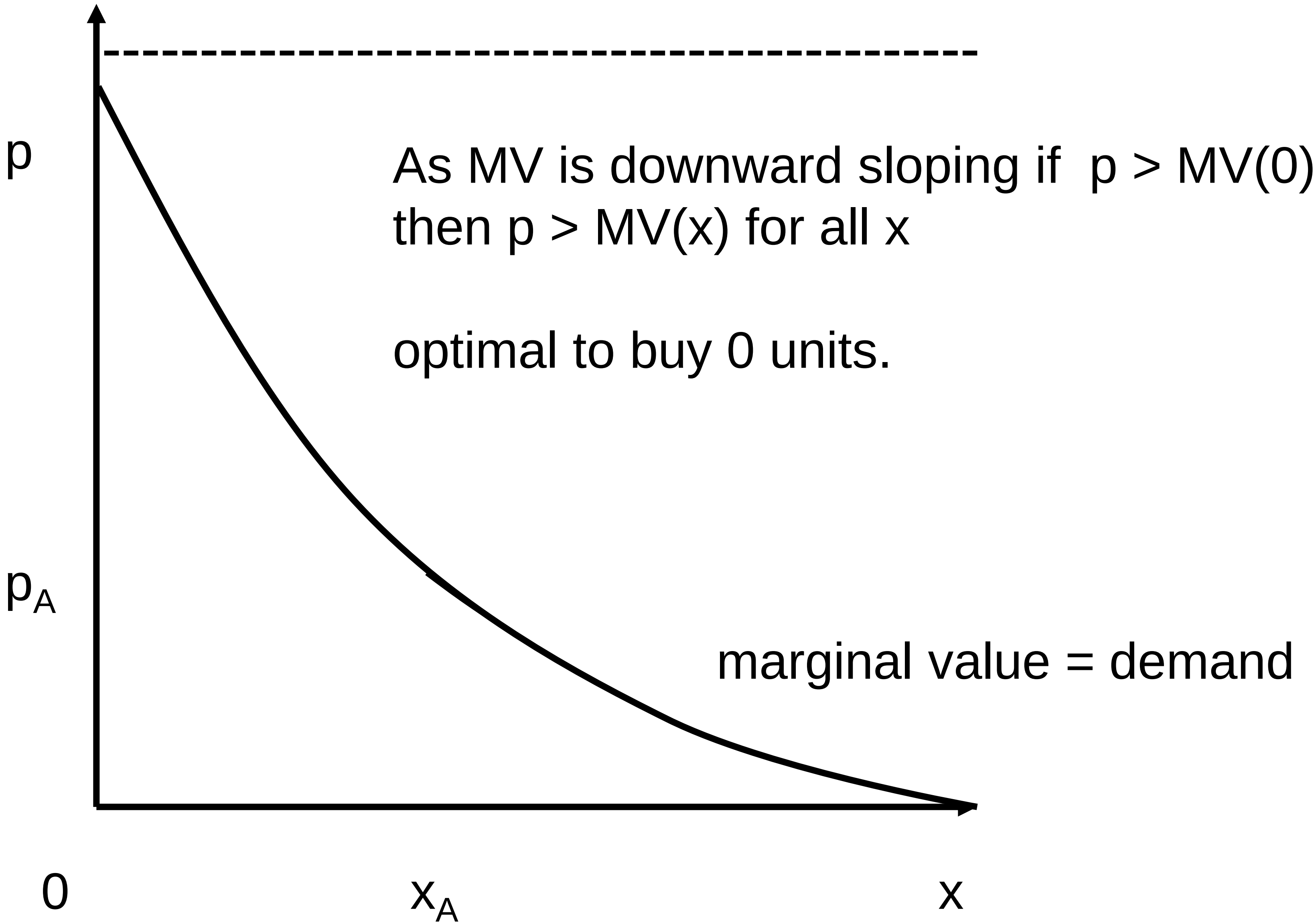
Why if MV= p_A at $x = x_A$ then x_A maximizes $V(x) – p_A x$?

If $x < x_A$ then MV > p_A, increasing x increases $V(x) – px$

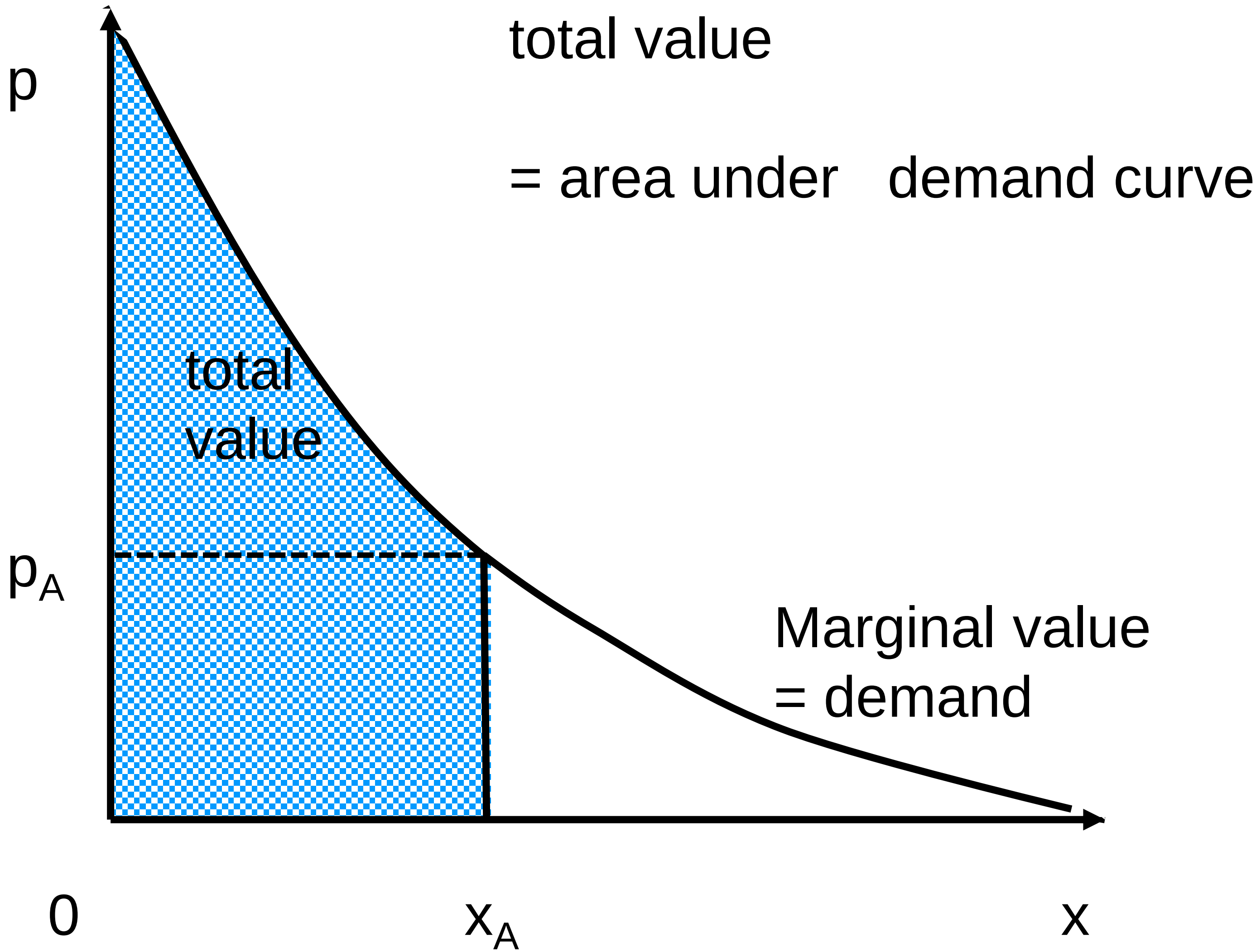
If $x > x_A$ then MV < p_A, increasing x decreases $V(x) - px$



Marshall's Theory with Calculus



Marshall's Theory with Calculus

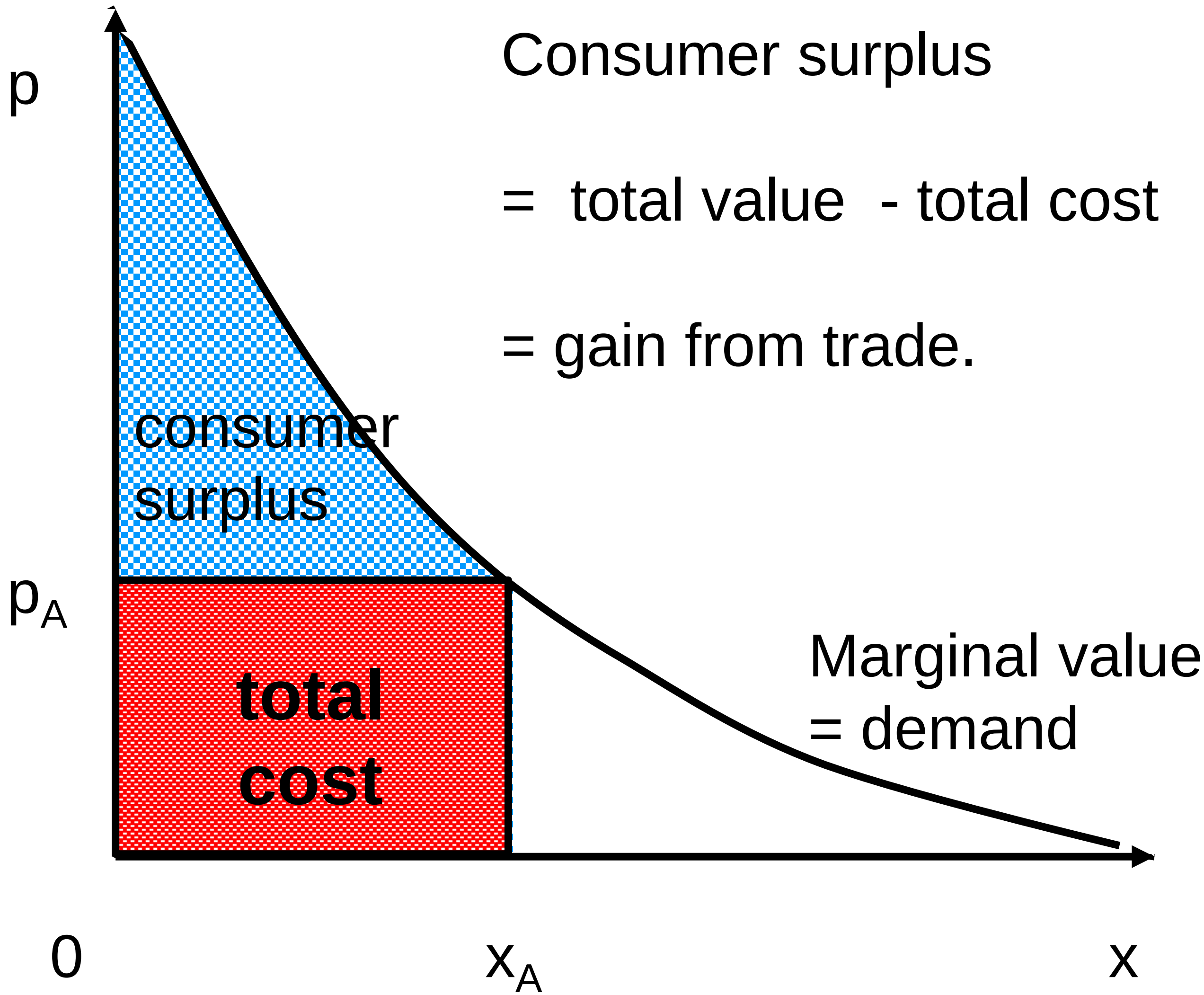


Marshall's Theory with Calculus

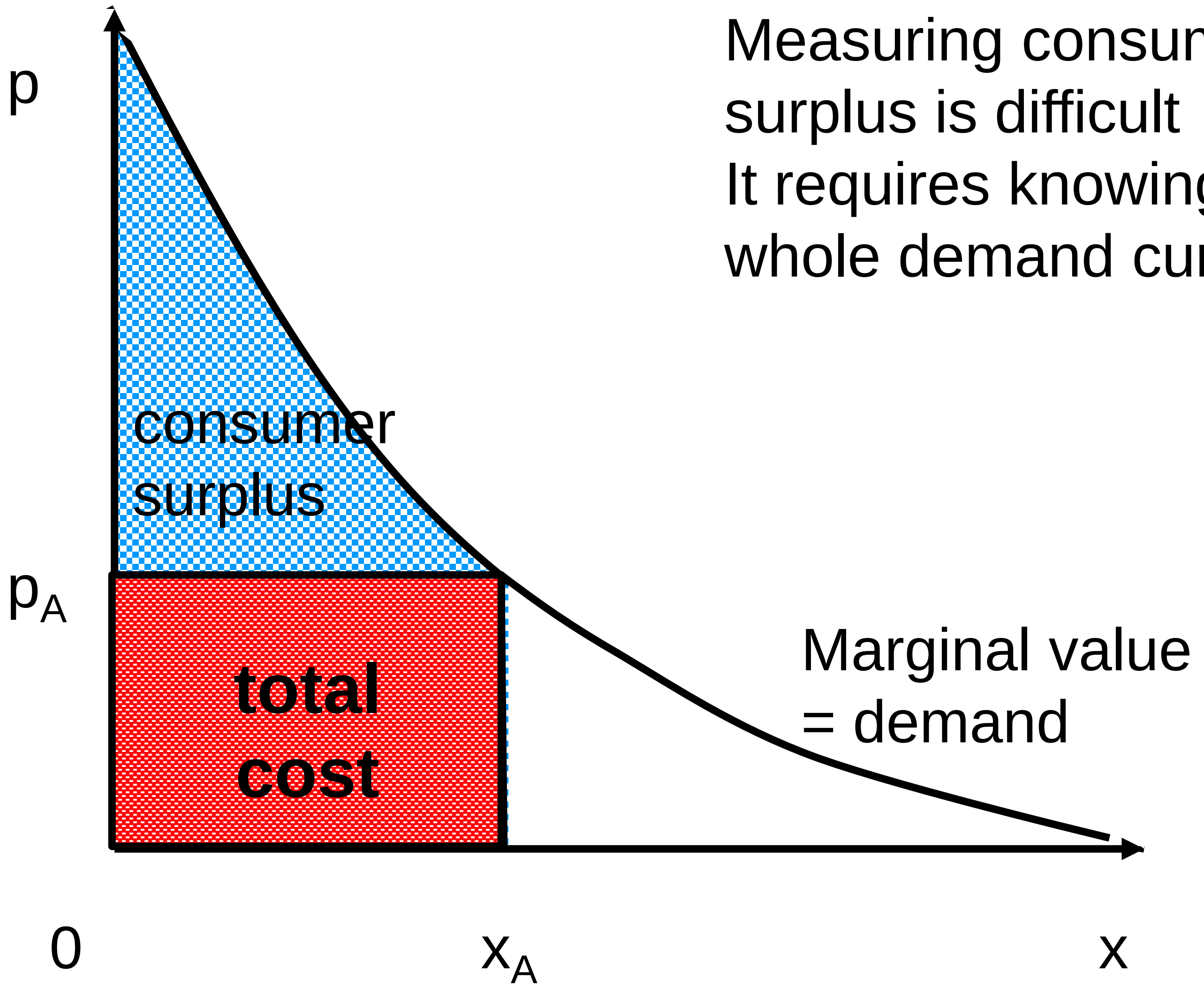
Consumer surplus

= total value - total cost

= gain from trade.

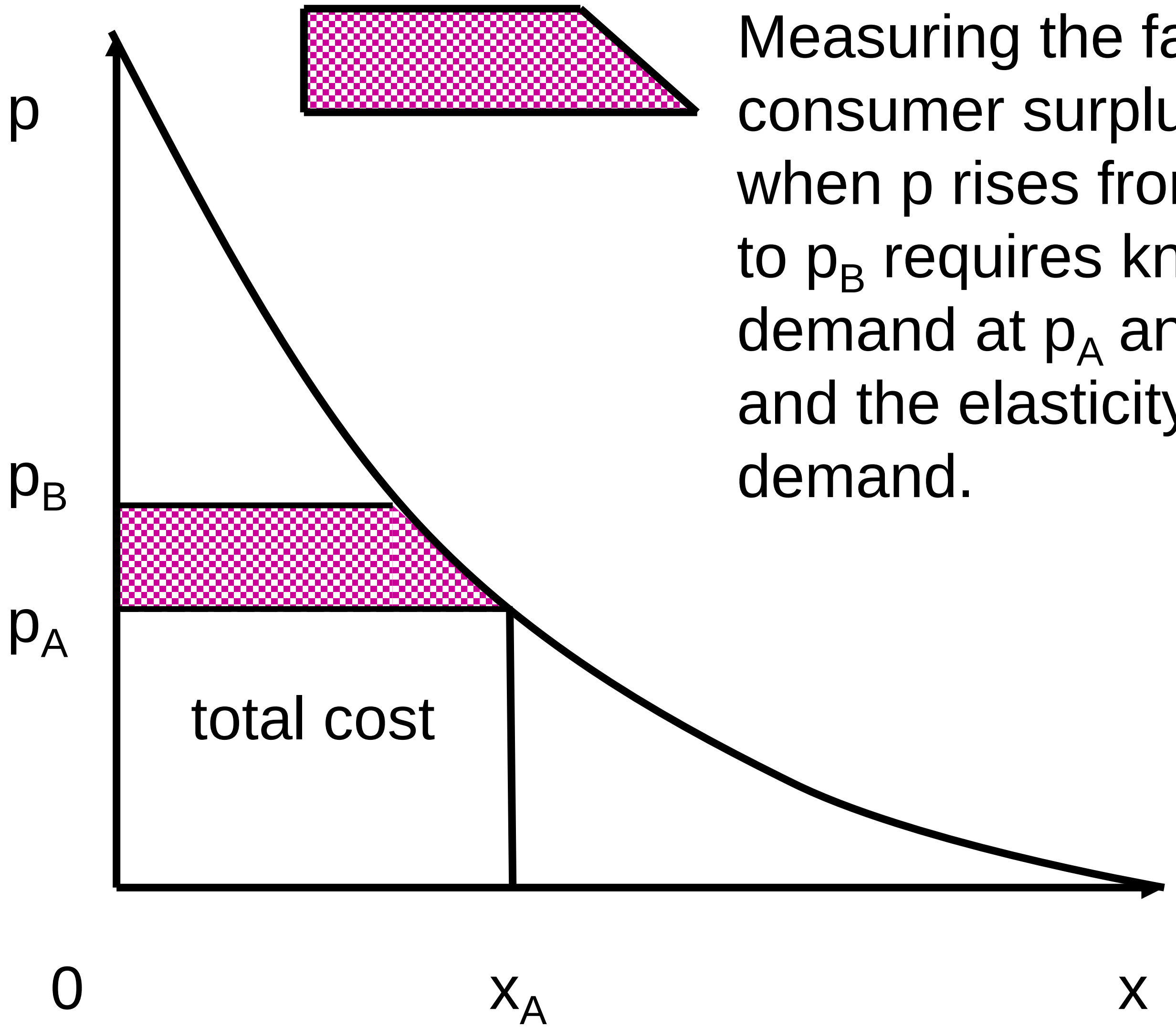


Marshall's Theory with Calculus



Measuring consumer surplus is difficult
It requires knowing the whole demand curve.

Marshall's Theory with Calculus

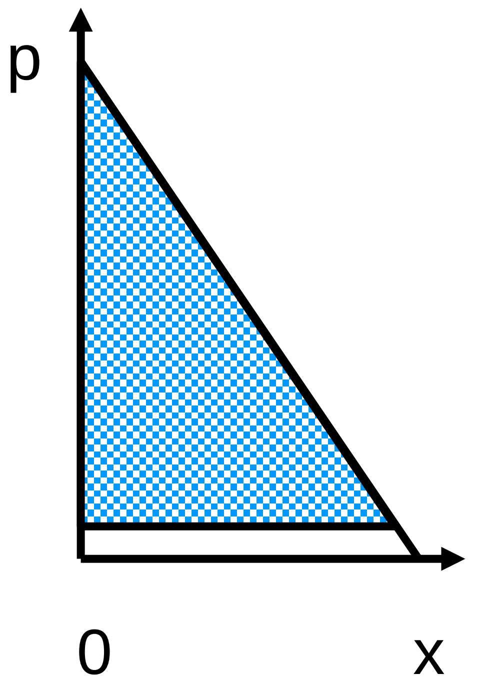


Measuring the fall in consumer surplus when p rises from p_A to p_B requires knowing demand at p_A and p_B and the elasticity of demand.

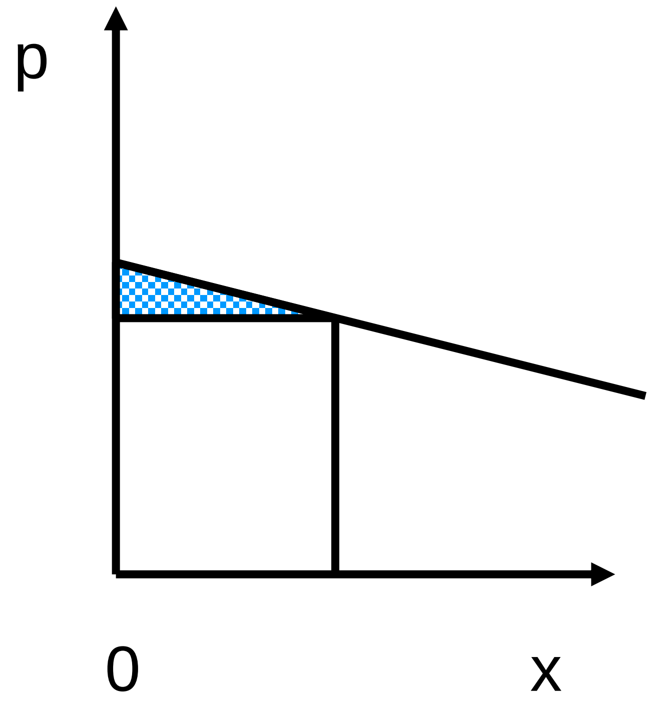
Diamond water paradox

Water is essential, diamonds are not

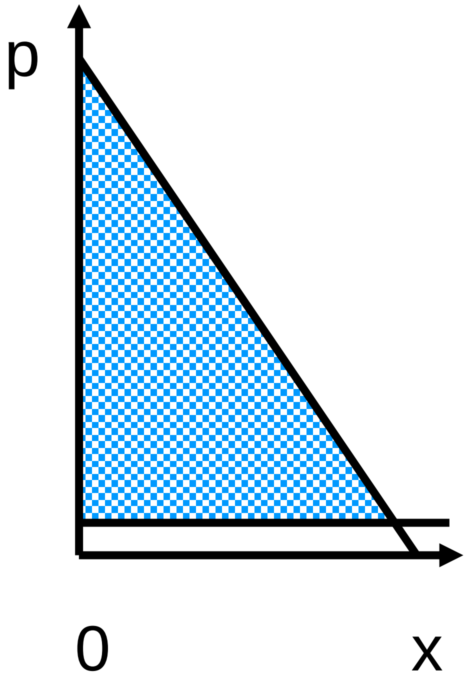
But diamonds have a much higher price than water.



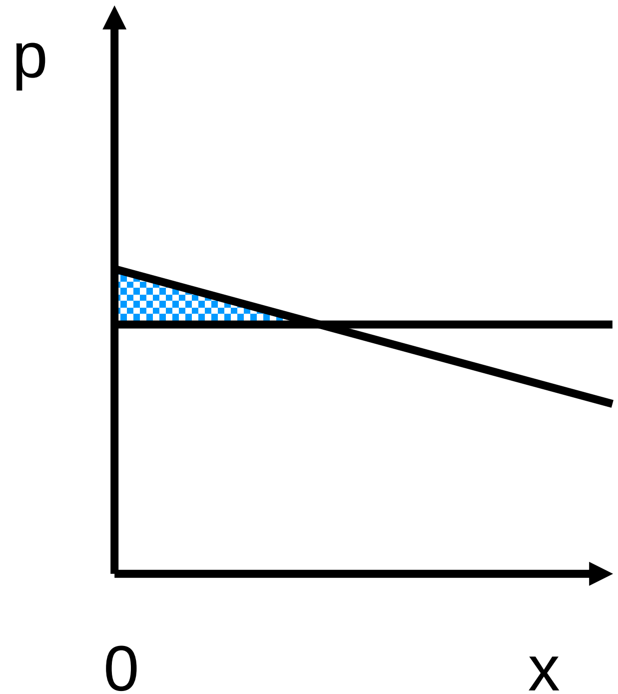
High consumer surplus low price, e.g. water



low consumer surplus high price, e.g. diamonds



Water, high total value,
low price, low marginal value,
large Consumer Surplus.



Diamonds, low total value,
high price, high marginal value,
small Consumer Surplus.

Marshall's theory explains why an essential good (water) has a much lower price than a luxury (diamonds).

What has Marshall achieved?

1. Explained the downward sloping demand curve.

2. Demonstrated that consumers gain from trade.

3. Giving a monetary measure of the gain from trade: consumer surplus – but this requires knowing the whole demand curve.

4. Explained why a good may have a low price but high total value,   price = marginal value, not total value.

   Diamond water paradox.

What are the difficulties with Marshall's model ?

- It does not distinguish between
  - changes in demand due to changes in the price of other goods & income
  - changes in demand due to changes in tastes, e.g. fashion.
- What is utility and can it be measured in money?
  - This is a big 19th century philosophical debate.

Preferences and utility

2. Hicks’ model of preferences and utility

General utility functions

Marshall has no model of how the utilities of two goods interact.

Move to utility depending on quantities of all goods,

Consuming x_1, x_2 ….x_n of goods 1,2…n gives utility

$u(x_1, x_2 \ldots . x_n)$ which is a number. This term n = 2.

Note the implicit assumption, utility depends on the goods you consume, can be extended to other people's consumption

There is no mention of human relationships in this theory.

Implications of the utility function 1: completeness

Because utility is a number for any two bundles of goods (x_{1A},x_{2A}) and (x_{1B},x_{2B}) one of these relationships must hold:

$u(x_{1A},x_{2A}) > u(x_{1B},x_{2B})$ the consumer **prefers** (x_{1A},x_{2A}) to (x_{1B},x_{2B})

$u(x_{1A},x_{2A}) = u(x_{1B},x_{2B})$ the consumer is **indifferent** between (x_{1A},x_{2A}) & (x_{1B},x_{2B}).

$u(x_{1A},x_{2A}) < u(x_{1B},x_{2B})$ the consumer **prefers** (x_{1B},x_{2B}) to (x_{1A},x_{2A})

Implications of the utility function 2: transitivity

Because utility is a number for any three bundles of goods (x_{1A},x_{2A}) and (x_{1B},x_{2B}) and (x_{1C},x_{2C})

If $u(x_{1A},x_{2A}) > u(x_{1B},x_{2B})$ and $u(x_{1B},x_{2B}) > u(x_{1C},x_{2C})$

then $u(x_{1A},x_{2A}) > u(x_{1C},x_{2C})$.

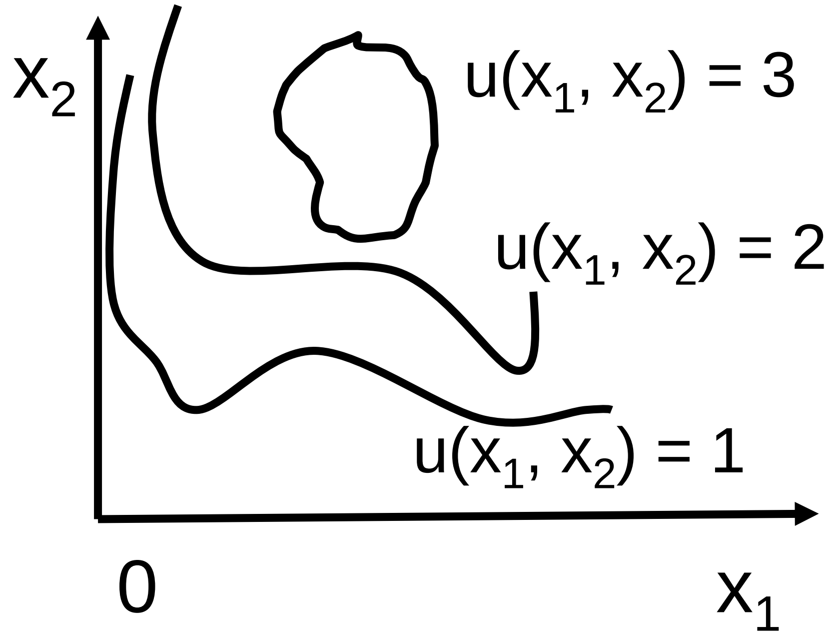
Implications of the utility function 3: indifference curves

All the points on an indifference curve have the same utility

so consumers are indifferent between two points on the same indifference curve.

There is an indifference curve through every point but we only show some of them.

Without further assumptions indifference curves can have a strange shape.



Hicks: "Value & Capital"

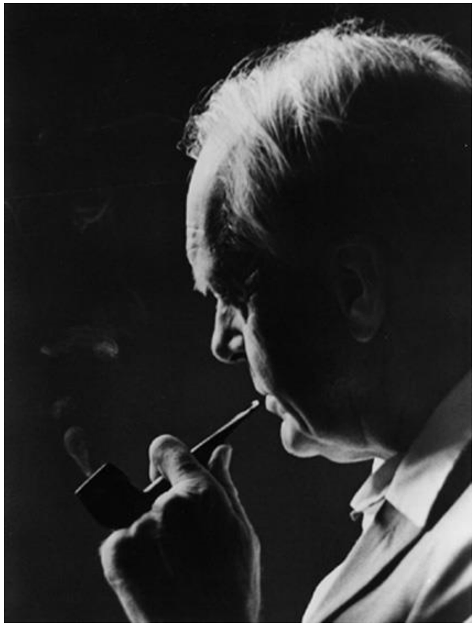
2nd edition, OUP 1939

Building on

Pareto (1909)

Edgeworth (1891)

Slutsky (1915)



Hicks worked with indifference curves

John Hicks,

1904 – 89

LSE 1926 to 1935

Nobel prize 1972

Source picture LSE website

3. Hicks on indifference curves:
Pareto's theory

"In order to determine the quantities of goods which an individual will buy at give prices,

Marshall's theory implies that we must know his utility surface; (i.e. utility function);

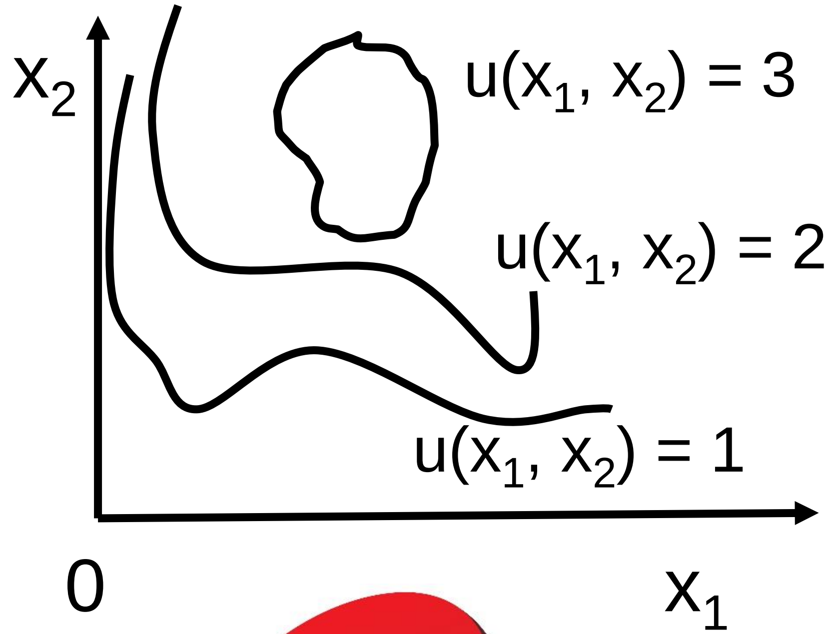
Pareto's theory only assumes that we must know his indifference map.
And that conveys less information than the utility surface.

It only tells us that that the individual prefers one particular collection of goods to another …..; it does not tell us *by how much* the first collection is preferred to the second."

Indifference curves

Does replacing 1, 2, 3 by 1, 4, 9 change preferences?

Does replacing 1, 2, 3 by 1, 30, 14 change preferences?

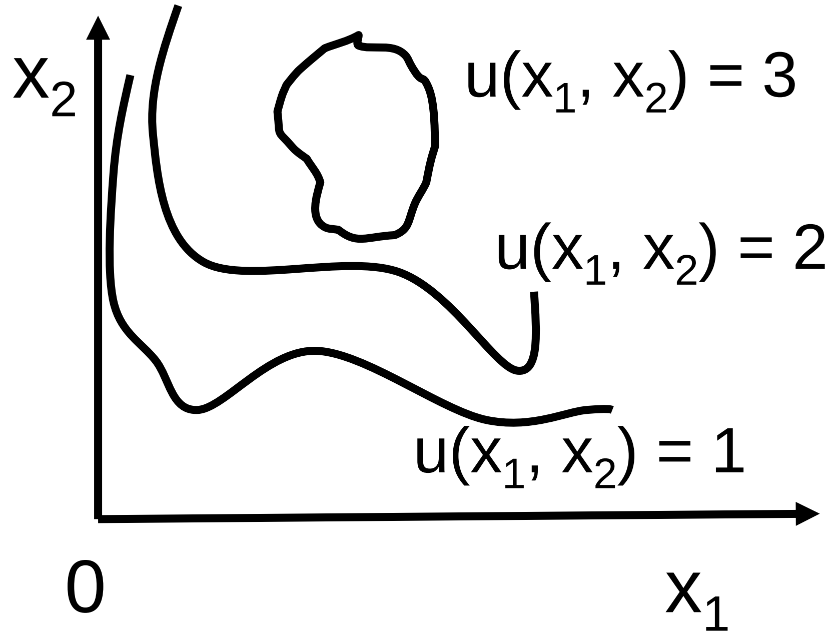


Indifference curves

Does replacing 1, 2, 3 by

10,100,1000, change

preferences? No



Does replacing 1, 2, 3 by

1, 30, 14 change preferences?

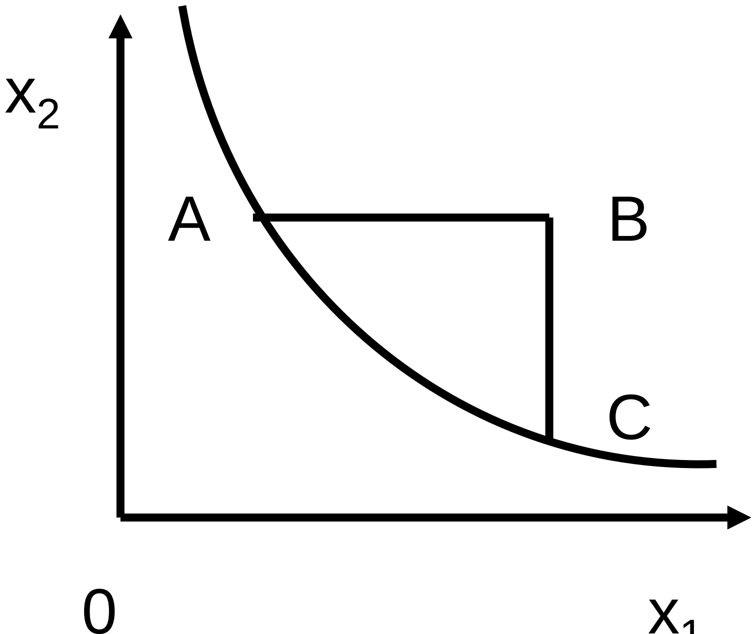
Yes

Hicks made two assumptions on the shape of indifference curve

- Nonsatiation, more is better.

- Decreasing marginal rate of substitution

Nonsatiation: more is better



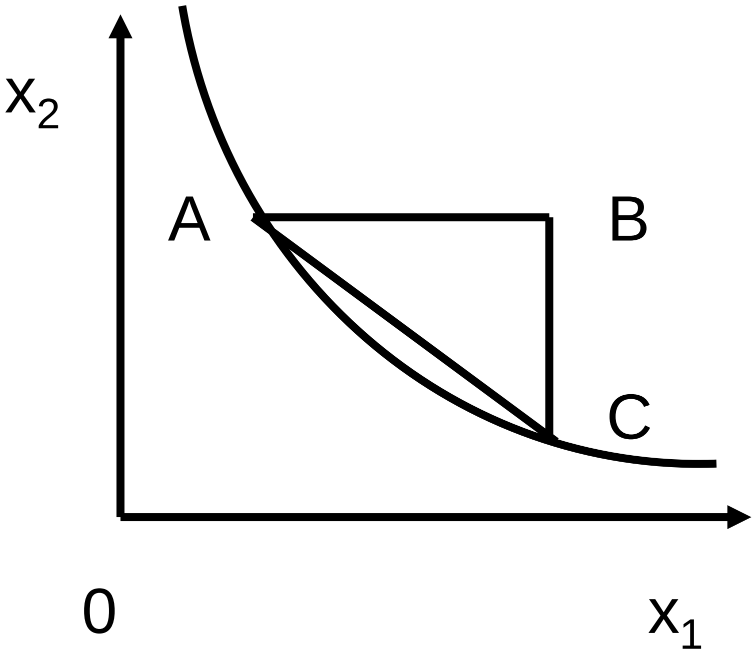
Consumers prefer more to less.

B is preferred to A because there is more good 1 and the same amount of good 2.

To get from B to C on the same indifference curve good 2 must be reduced.

Nonsatiation implies downward sloping indifference curves.

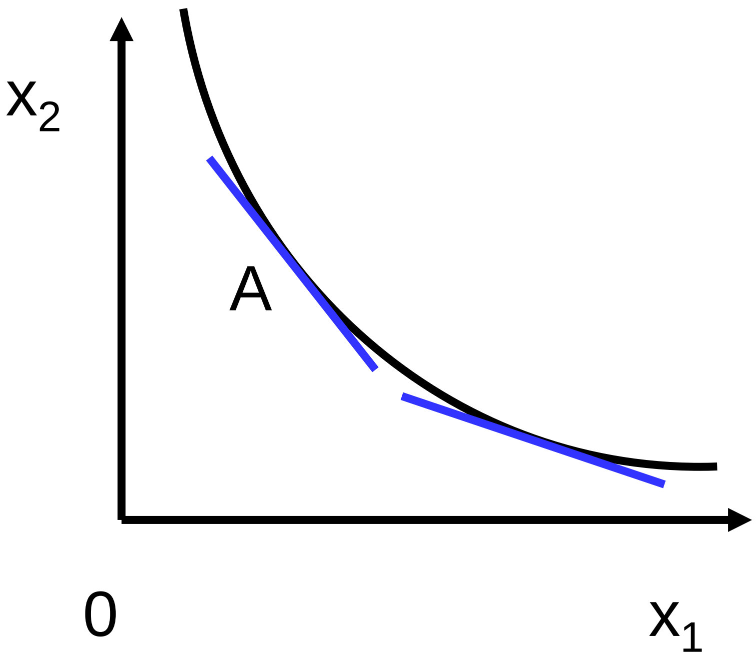
Hicks introduced the term "Marginal Rate of Substitution", MRS



$\frac{BC}{AB}$ = - slope of AC

= average amount of good 2 needed to compensate for loosing 1 unit of good 1.

Hicks assumed decreasing MRS



MRS = - gradient of indifference curve

Hicks assumes that the MRS decreases along an indifference curve as x_1 increases.

With more x_1 the consumer requires less extra x_2 to make up for loosing a unit of x_1.

Summary

- Marshall started his analysis with a utility functions.
- Hicks dropped utility functions and started his analysis with indifference curves.
- The modern theory starts with preferences

Preferences and utility

3. Modern assumptions on preferences and utility

Modern assumptions on preferences

1. Completeness
2. Transitivity
3. Continuity
4. Nonsatiation
5. Convexity

For each of these assumption I give

- *a definition*
- *an intuitive explanation*
- *a critical discussion.*

Modern assumptions on preferences: completeness

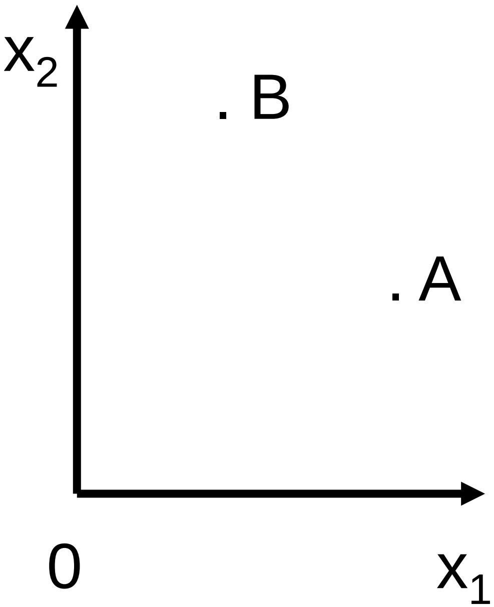
Modern Assumption 1: Completeness

If a consumer is choosing between two bundles A and B one of the following possibilities holds

she prefers A to B

she prefers B to A

she is indifferent between A and B.



This term we work with 2 goods so consumption bundles can be shown in a diagram.

Choices over Chinese vegetables

si-gua

mao-gua

jay-lan

Consumers may find it impossible to rank some options, or do so in a way that is in some sense wrong for them.

Decision making takes time and effort, we are often

- ignorant
- uncertain
- unable to understand what a product is and does
- subject to systematic biases

This standard model is not based on psychological research.

Behavioural economics is based on psychological research.

Modern assumptions on preferences: transitivity

Modern Assumption 2: Transitivity

Transitivity if A is preferred to B and B to C then A is preferred to C.

Is the relationship "A is taller than B" is transitive?

Is the relationship "team A beat team B last time they played" transitive?

Modern Assumption 2: Transitivity

Transitivity if A is preferred to B and B to C then A is preferred to C.

Is the relationship "A is taller than B" is transitive?

yes

Is the relationship "team A beat team B last time they played" transitive?

Modern Assumption 2: Transitivity

Transitivity if A is preferred to B and B to C then A is preferred to C.

Is the relationship "A is taller than B" is transitive?

yes

Is the relationship "team A beat team B last time they played" transitive?

no

Premier League 2015/16

Leicester beat Chelsea

Chelsea beat Arsenal

Arsenal beat Leicester

Transitivity would imply that Leicester beat Arsenal.

The relationship beat in sport is transitive.

Premier League 2015/16

Leicester beat Chelsea

Chelsea beat Arsenal

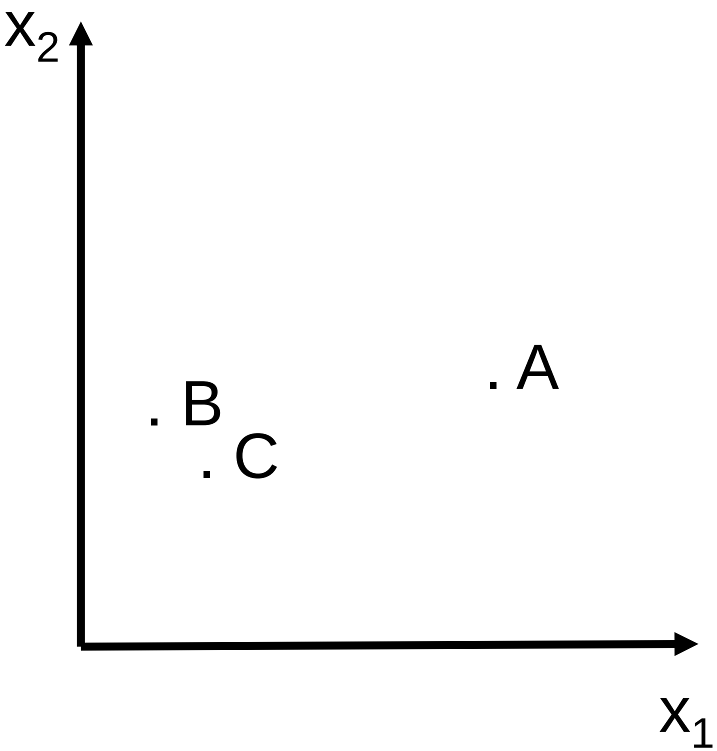
Arsenal beat Leicester

Transitivity would imply that Leicester beat Arsenal.

The relationship beat in sport is not transitive.

Modern assumptions on preferences: continuity

Modern Assumption 3: Continuity



If A is preferred to B

& B is very close to C

then A is preferred to C.

This can be stated much more precisely but not in this course.

Summary: Modern theory starts with a preference relation and assumes

1. **Completeness**, if A and B are two bundles of goods then
   - either A is preferred to B,
   - or A and B are indifferent
   - or B is preferred to A
2. **Transitivity** if A is preferred to B and B to C then A is preferred to C
3. **Continuity** if A is preferred to B and C is close to B then A is preferred to C.

The completeness, transitivity and continuity assumptions imply

- preferences can be represented by a utility function and indifference curves.

- Take this on trust.

- Utility is **ordinal** i.e. different utility functions can represent the same preferences.

Consumer theory: ordinal utility

Indifference curves

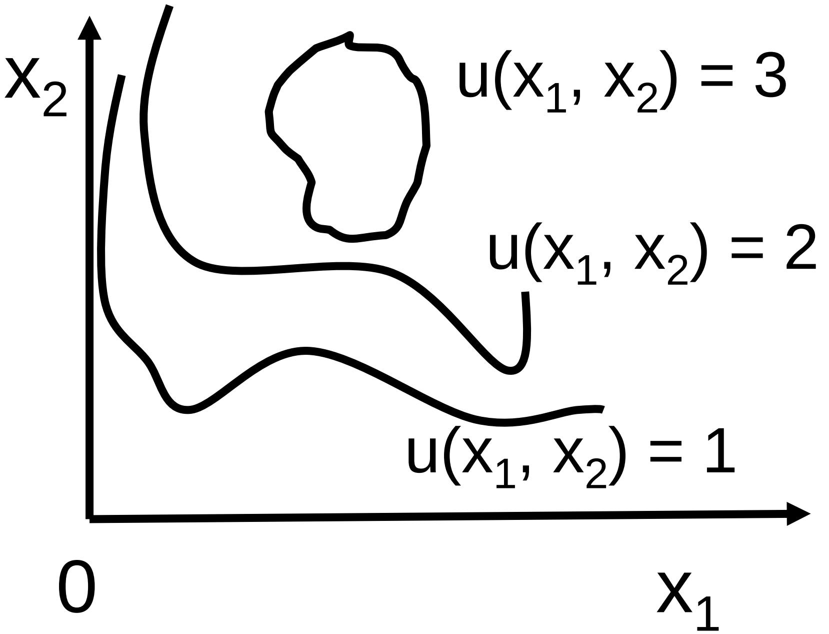
Replacing 1, 2, 3 by 1, 4, 9,

does not change preferences.

Replacing 1, 2, 3 by

1, 30, 14 **does change**

preferences



Different utility functions representing the same preferences

- Suppose utility $u(x_1,x_2) = x_1^{3/4} x_2^{1/2}$
- If $x_1 > 0$ and $x_2 > 0$ $u > 0$
- Then as u^2 is an increasing function of u (for u>0)

  if $v = u^2$ then

  $v(x_1,x_2) = (u(x_1,x_2))^2 = x_1^{3/2} x_2$ represents the same preferences as $u(x_1,x_2)$
- This is because the order of numbers attached to indifference curves does not change.

Different utility functions representing the same preferences

- Suppose utility $u(x_1,x_2) = x_1^{3/4} x_2^{1/2}$
- If $x_1 > 0$ and $x_2 > 0$ $u > 0$
- Then as ln u is an increasing function of u (for u>0)

  if $w = \ln u$ then

  $w(x_1,x_2) = \ln(u(x_1,x_2)) = (3/4) \ln x_1 + (1/2) \ln x_2$

  represents the same preferences as $u(x_1,x_2)$

- This is because the order of numbers attached to indifference curves does not change.

Different utility functions representing the same preferences

- Suppose utility $u(x_1,x_2) = \min[2x_1, 3x_2]$
- If $x_1 > 0$ and $x_2 > 0$ $u > 0$
- Then as 2u is an increasing function of u

  if $w = 2u$ then

  $z(x_1,x_2) = 2\min[2x_1, 3x_2] = \min[4x_1, 6x_2]$

  represents the same preferences as $u(x_1,x_2)$
- This is because the order of numbers attached to indifference curves does not change.

Different utility functions representing the same preferences

- If f(u) is an increasing function of u
- Then f(u(x)) represents the same preferences as u(x).
- Utility is **ordinal not cardinal.**

Modern assumptions on preferences: nonsatiation

More is better.

Nonsatiation in Nicholson, Snyder, Stewart

Section on economic goods

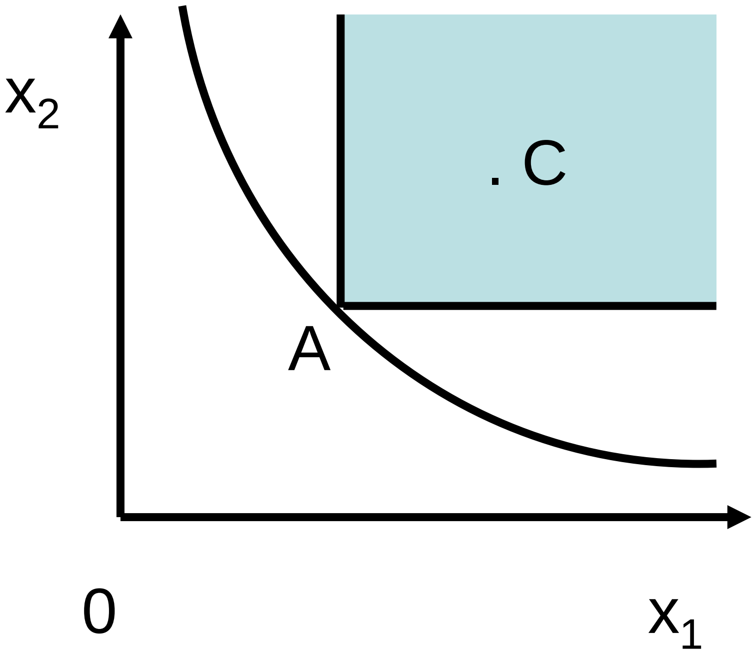
> "… the variables are taken to be "goods", i.e. whatever quantities they represent we assume that more of any particular good …. is preferred to less."

A consumer prefers having more of either good to having less.

If $x_{1B} > x_{1A}$ then (x_{1B},x_{2A}) is preferred to (x_{1A},x_{2A})

If $x_{2B} > x_{2A}$ then (x_{1A},x_{2B}) is preferred to (x_{1A},x_{2A})

Nonsatiation in the indifference curve diagram



Nonsatiation means that any point such as C inside or on the boundary of the shaded area is preferred to A.

Here starting from A increasing x_1 and/or increasing x_2 increases utility.

Nonsatiation implies indifference curves slope downwards

5. Checking the nonsatiation assumption

If $\dfrac{\partial u}{\partial x_1} > 0, \quad \dfrac{\partial u}{\partial x_2} > 0$

then the nonsatiation assumption is satisfied

e.g. if $u = x_1^{2/5} x_2^{3/5}$

$$\frac{\partial u}{\partial x_1} = \frac{2}{5} x_1^{-3/5} x_2^{3/5} > 0,$$

$$\frac{\partial u}{\partial x_2} = \frac{3}{5} x_1^{2/5} x_2^{-2/5} > 0$$

so the nonsatiation assumption is satisfied.

5. Checking the nonsatiation assumption

If $$\frac{\partial u}{\partial x_1} > 0, \qquad \frac{\partial u}{\partial x_2} > 0$$

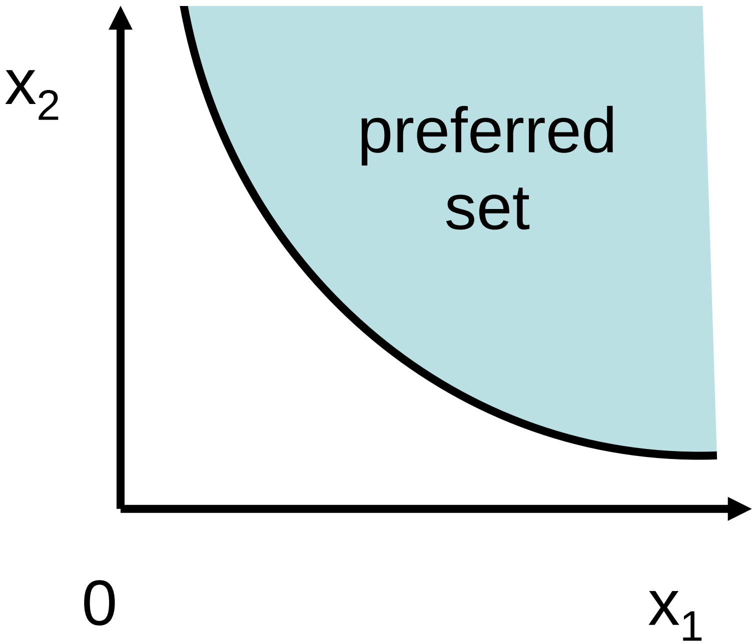
then increasing x_1 or x_2 increases utility.

The indifference curve slopes downwards.

The preferred set is above the indifference curve.

Nonsatiation is satisfied.

Nonsatiation in the indifference curve diagram

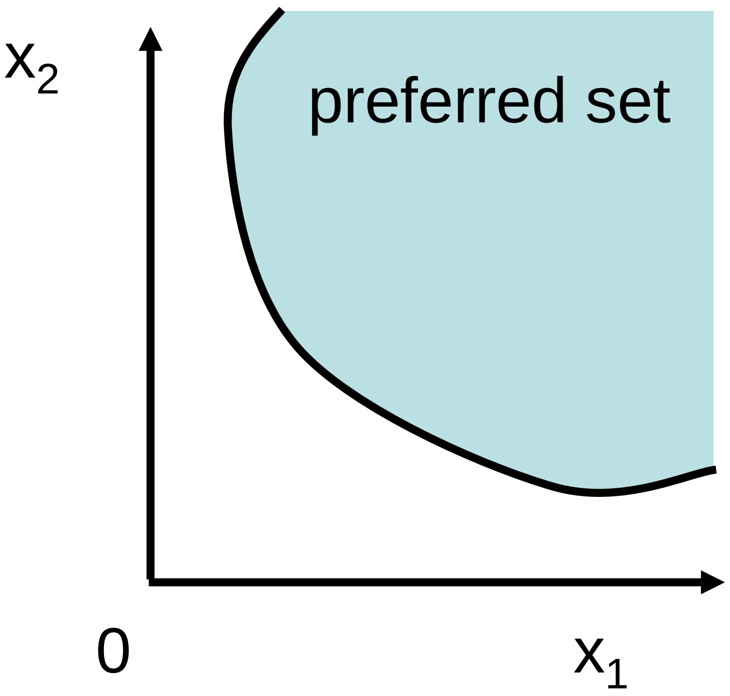


Nonsatiation implies that points above an indifference curve are preferred to points on the indifference curve.

These points are in the preferred set.

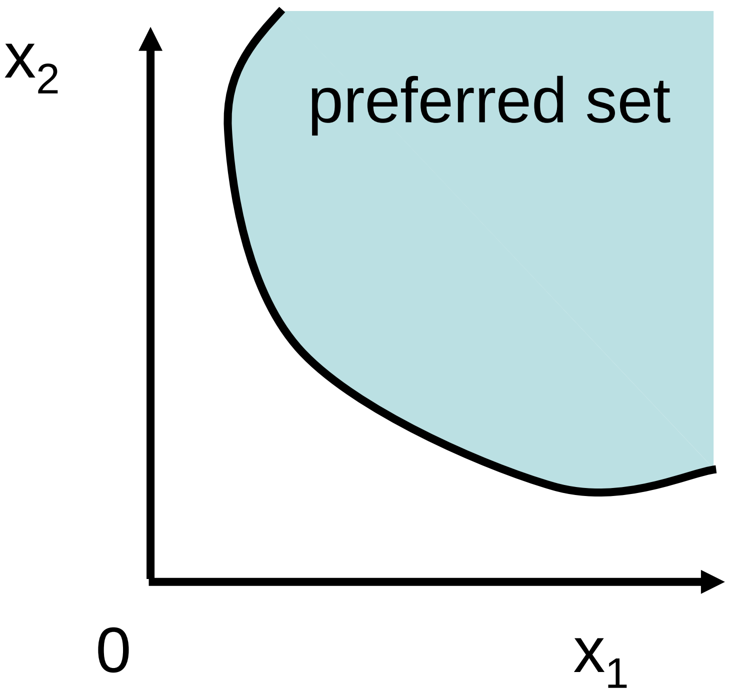
Nonsatiation implies downward sloping indifference curves.

Is nonsatiation satisfied here?



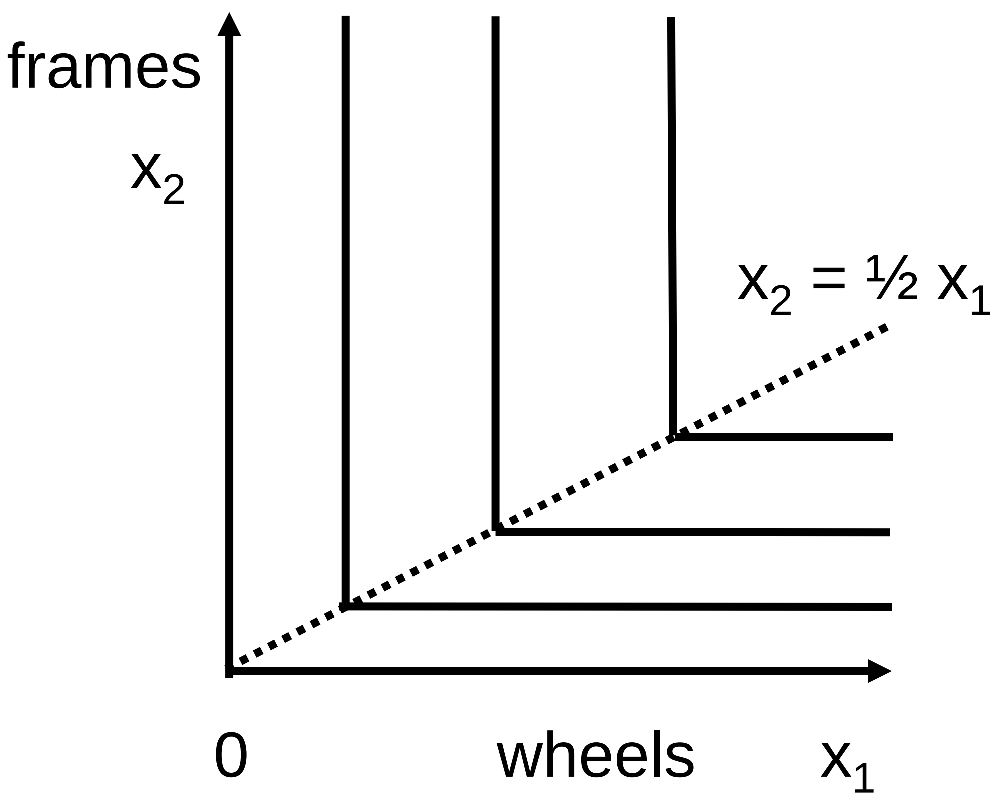
Is nonsatiation satisfied here?

NO



Drawing indifference curves like this is an easy mistake.

Perfect complements utility: requires a different treatment of nonsatiation.



$u(x_1,x_2) = \min(\tfrac{1}{2} x_1, x_2)$

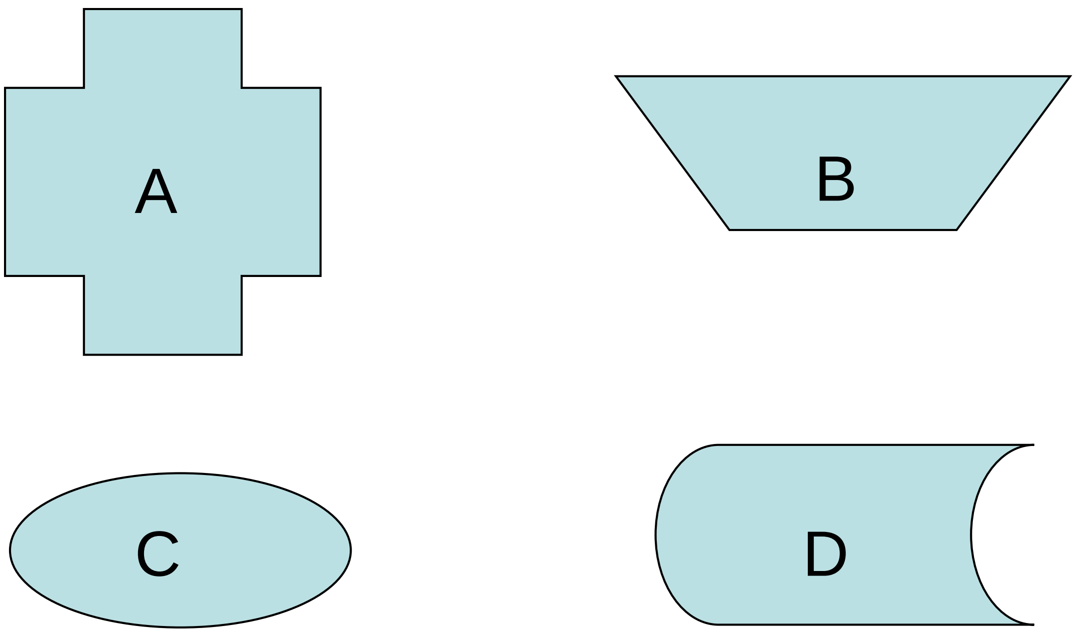
x_1 bicycle wheels,
x_2 bicycle frames

Modern assumptions on preferences: convexity

Modern Assumption 5: Convexity
The preferred set is convex.

Mathematically a set is **convex** if any straight line joining two points in the set lies in the set.

Which of these sets are convex?



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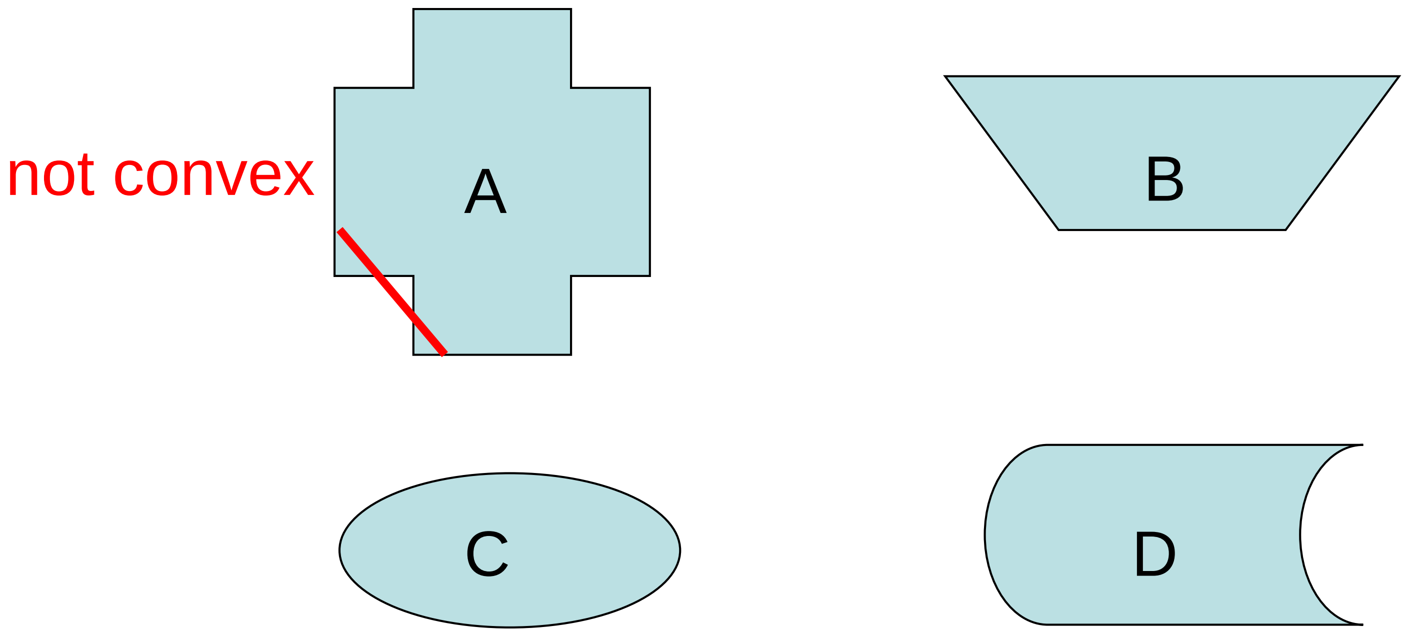

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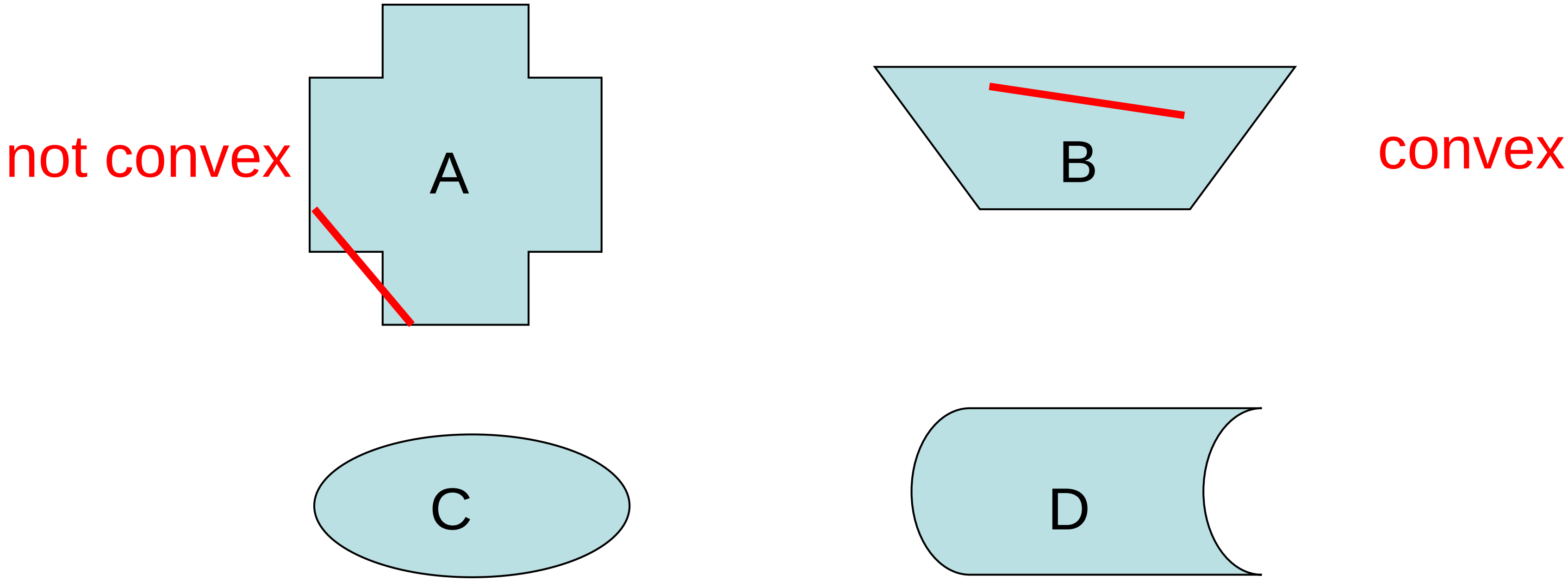

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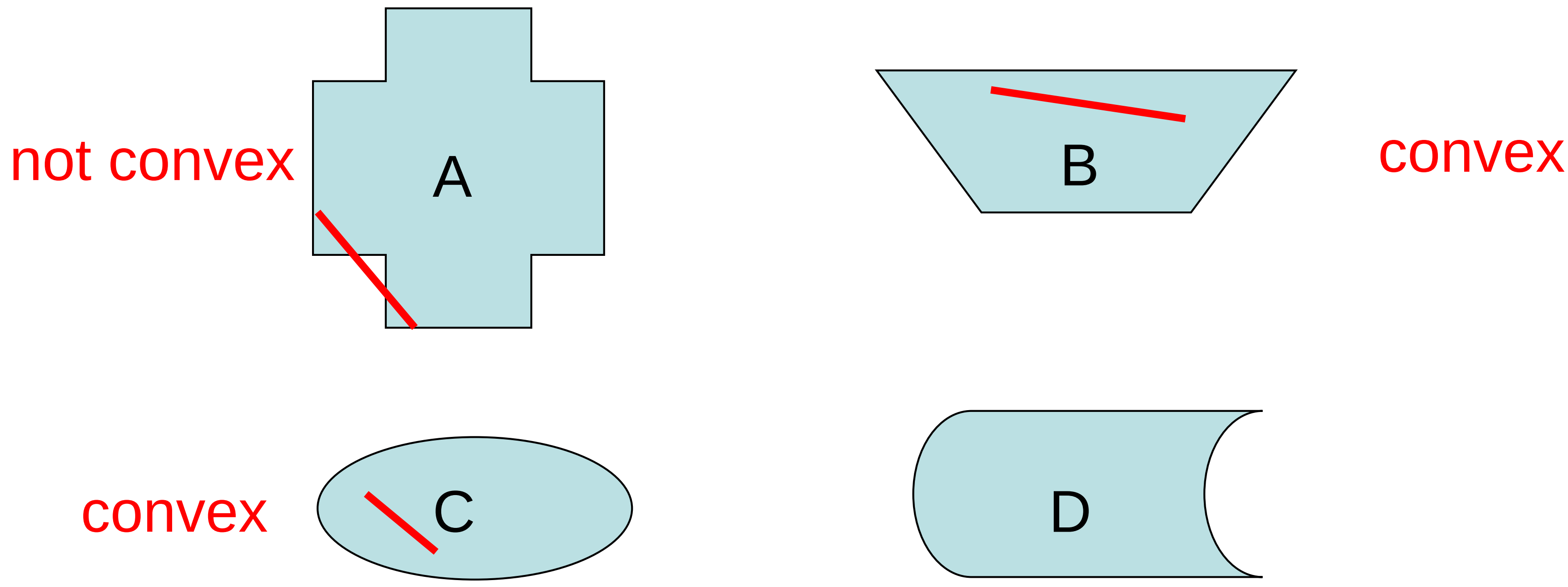

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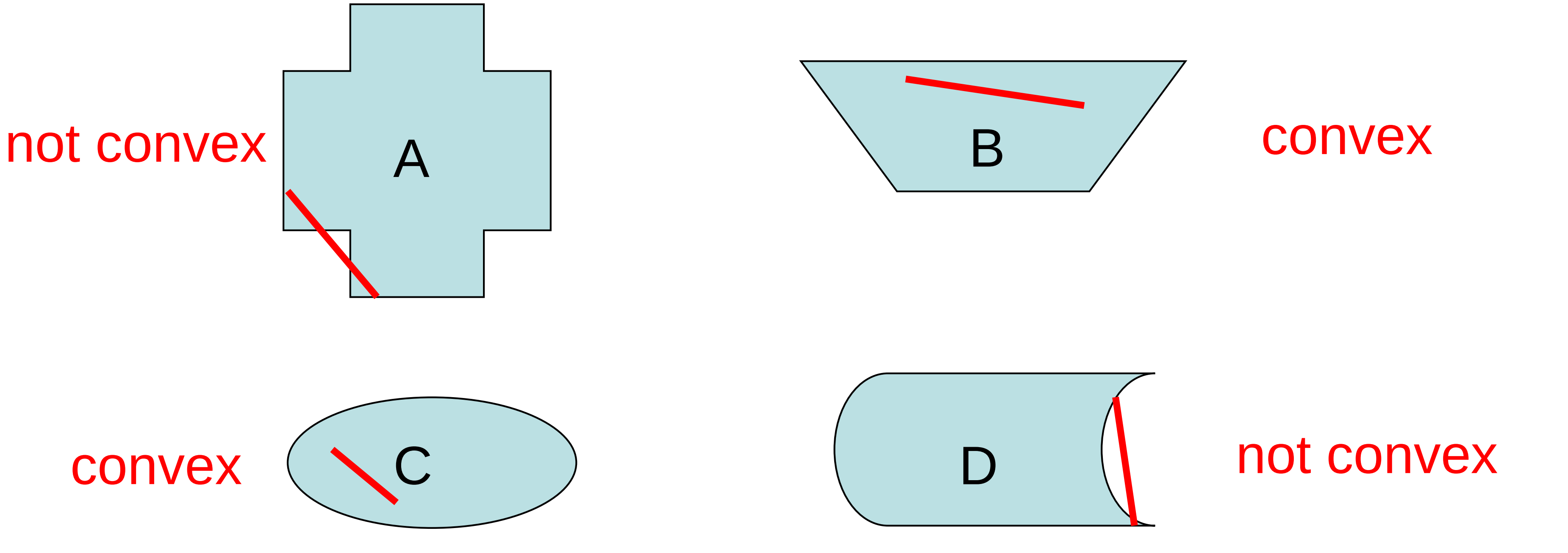

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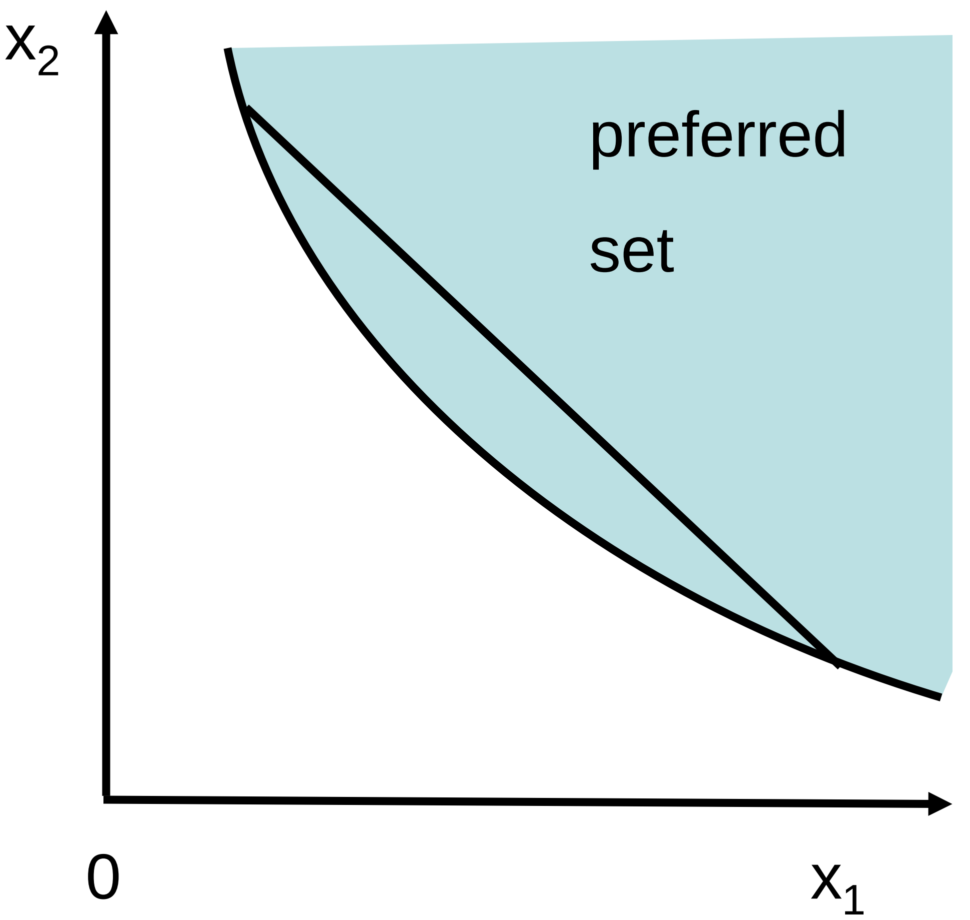
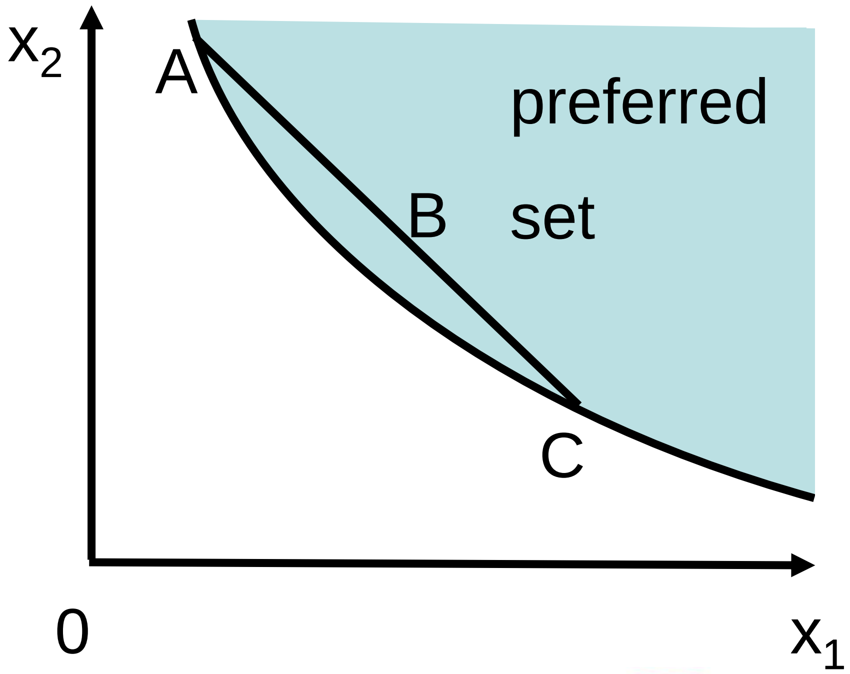


Interpreting the convexity assumption

A and C are indifferent.



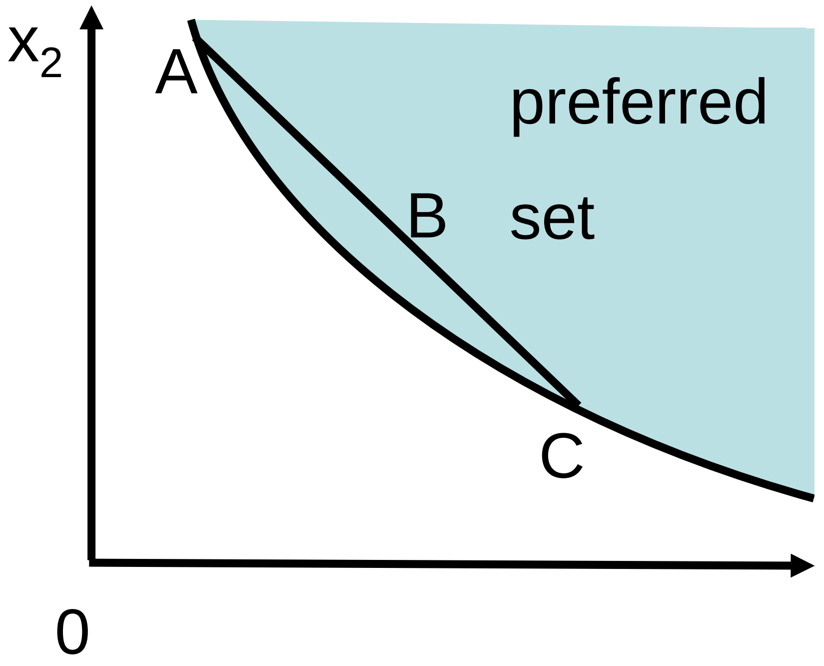
B is the of A and C

Convexity implies that B is preferred to

Convexity means preference

Interpreting the convexity assumption

A and C are indifferent.



B is the average of A and C

Convexity implies that B is preferred to

Convexity means preference

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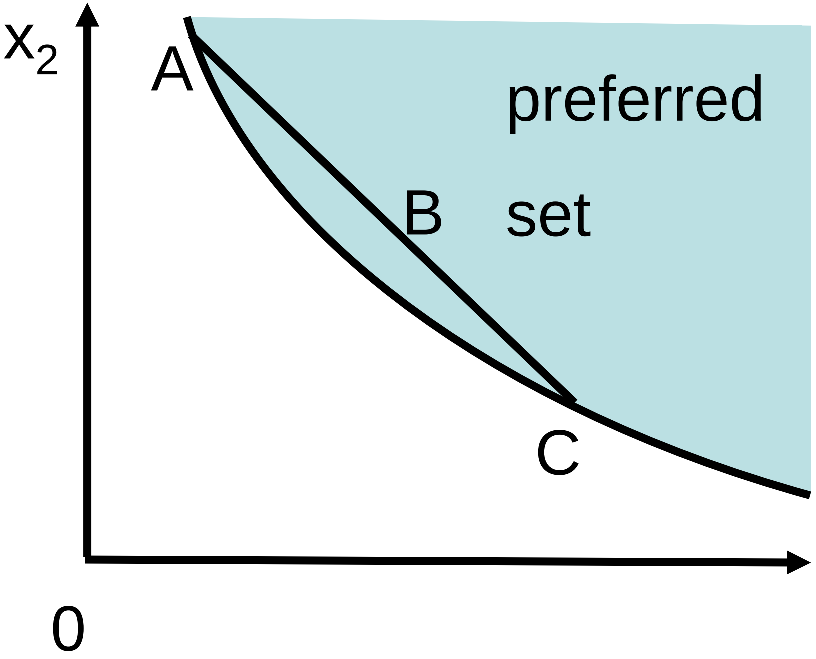
A and C are indifferent.



B is the average of A and C

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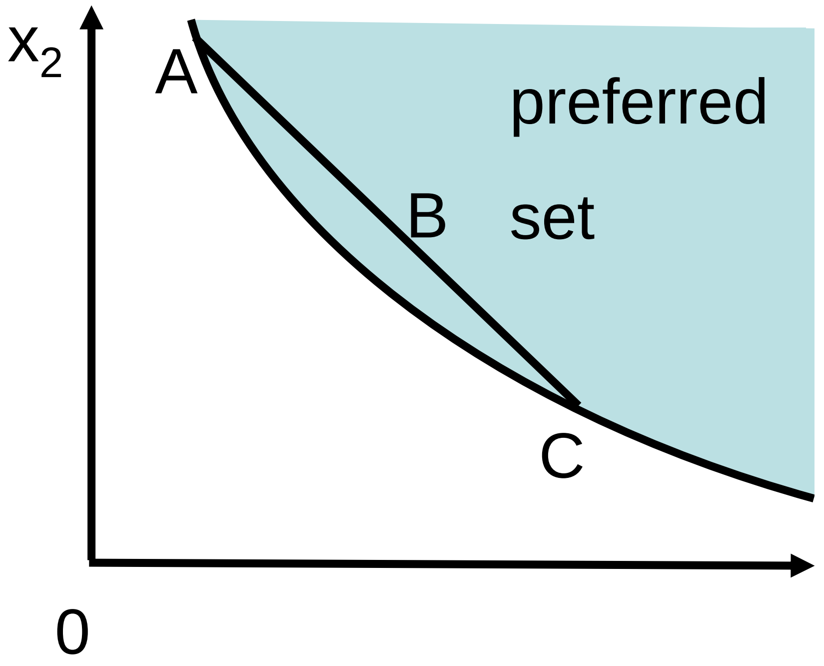
Convexity means preference

Interpreting the convexity assumption

A and C are indifferent.

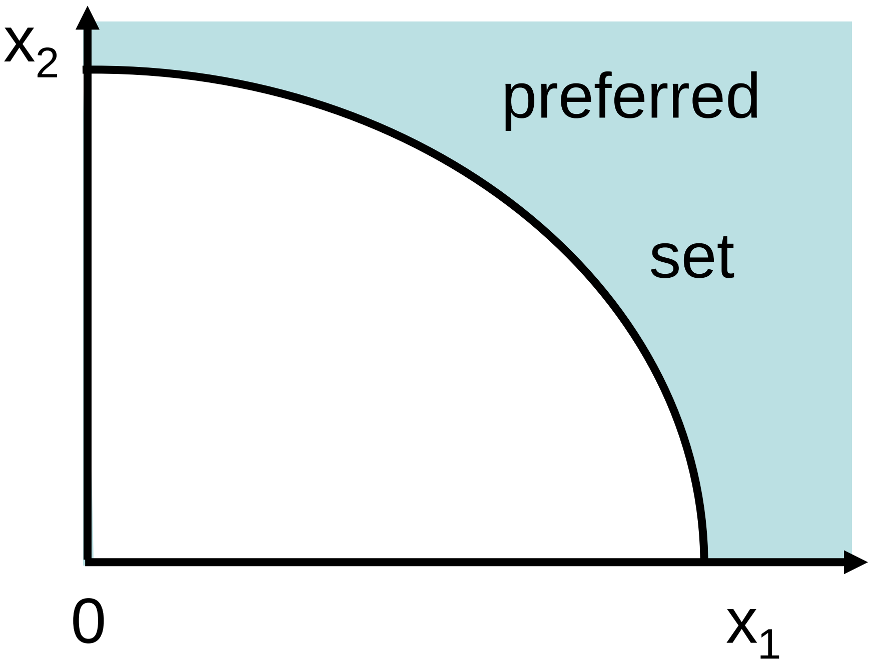


B is the average of A and C

Convexity implies that B is preferred to A and C.

Convexity means preference for averages.

Nonconvex preferences with nonsatiation



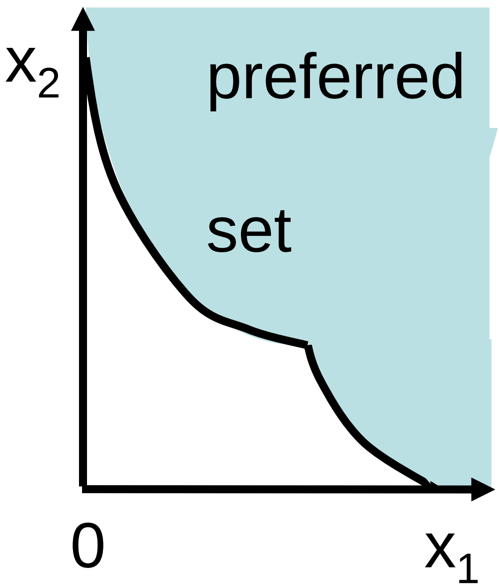
possible examples of nonconvex preferences

tea & coffee consumed simultaneously

alcohol & drugs

Do these preferences satisfy convexity and nonsatiation?

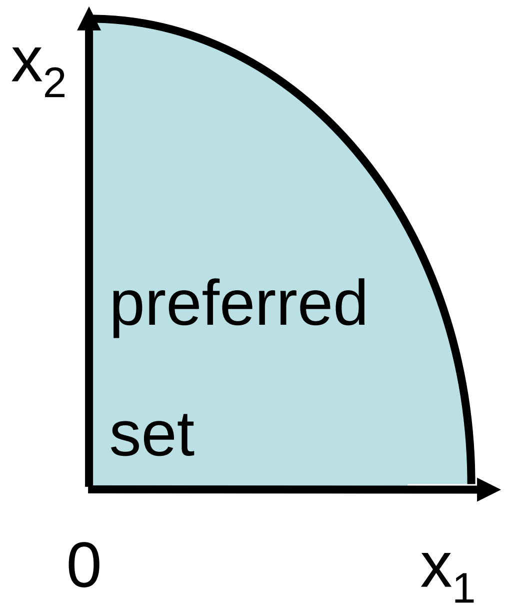
Figure 1



convexity

nonsatiation

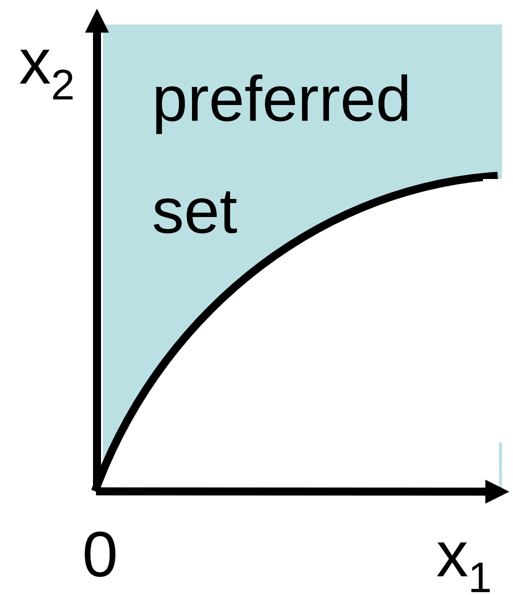
Figure 2



convexity

nonsatiation

Figure 3



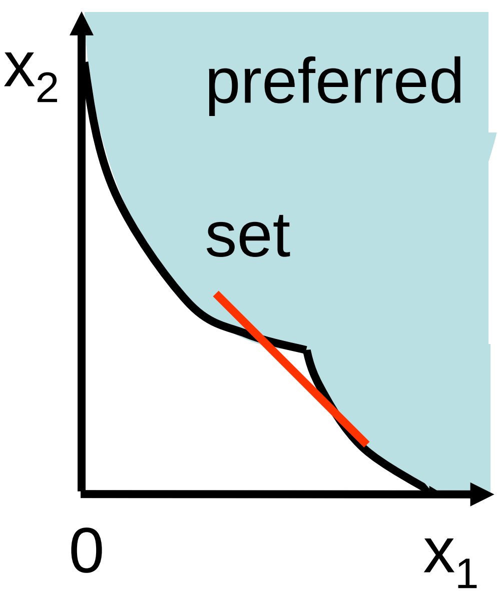
convexity

nonsatiation

TP

Do these preferences satisfy convexity and nonsatiation?

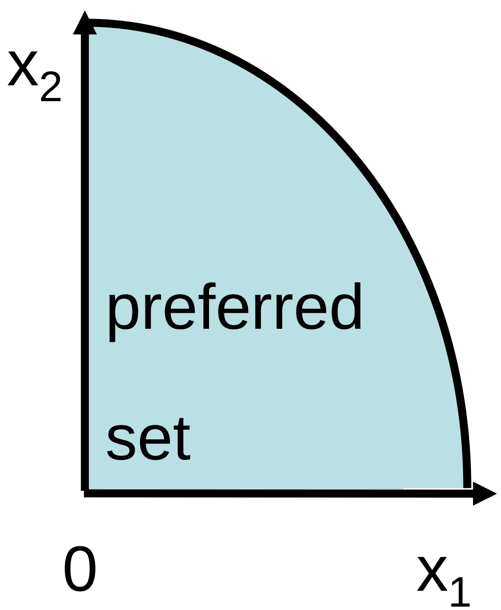
Figure 1



convexity no

nonsatiation yes

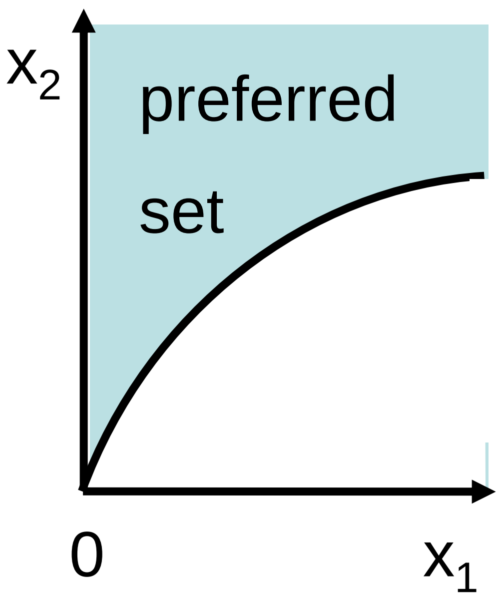
Figure 2



convexity

nonsatiation

Figure 3



convexity

nonsatiation

Do these preferences satisfy convexity and nonsatiation?

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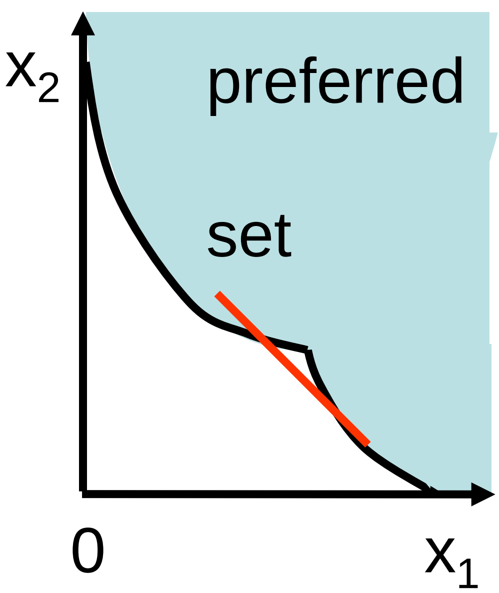
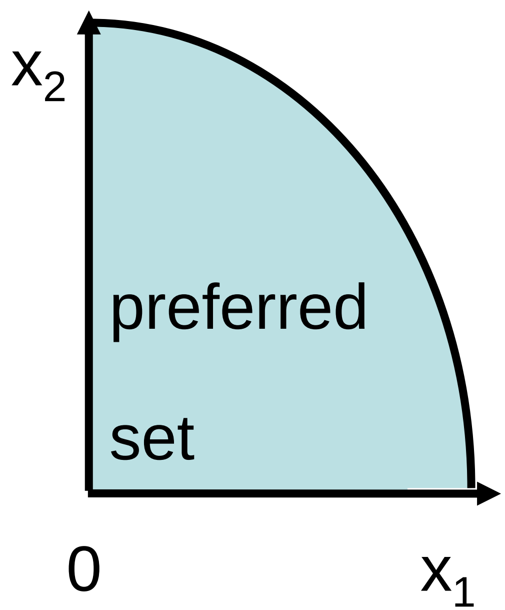


convexity no

nonsatiation yes

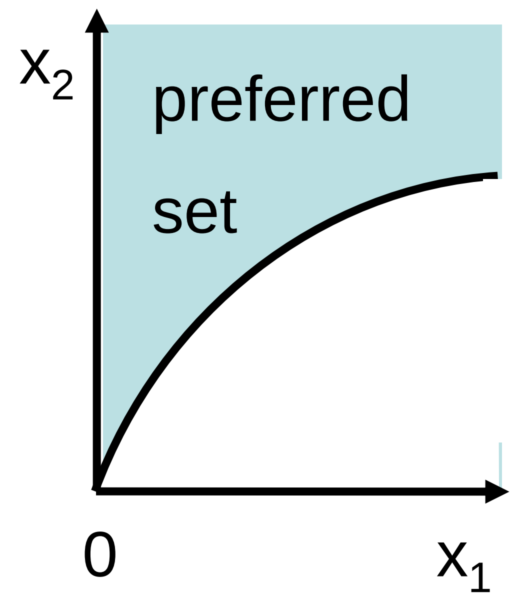
Figure 2



convexity yes

nonsatiation no

Figure 3

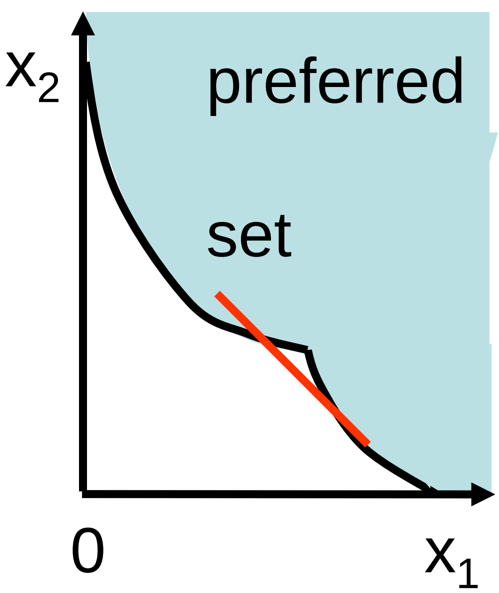


convexity

nonsatiation

Do these preferences satisfy convexity and nonsatiation?

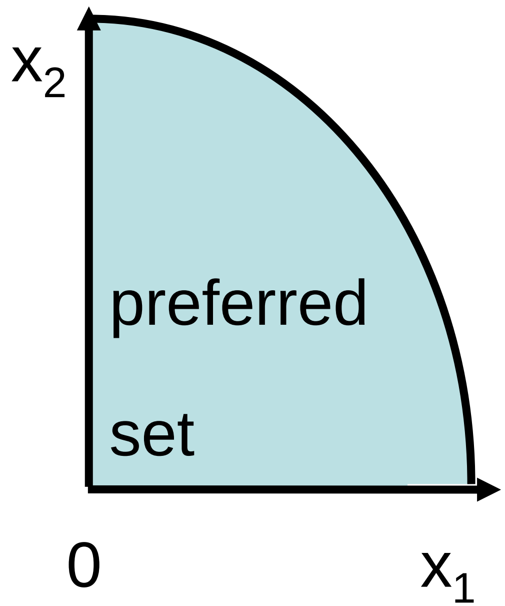
Figure 1



convexity no

nonsatiation yes

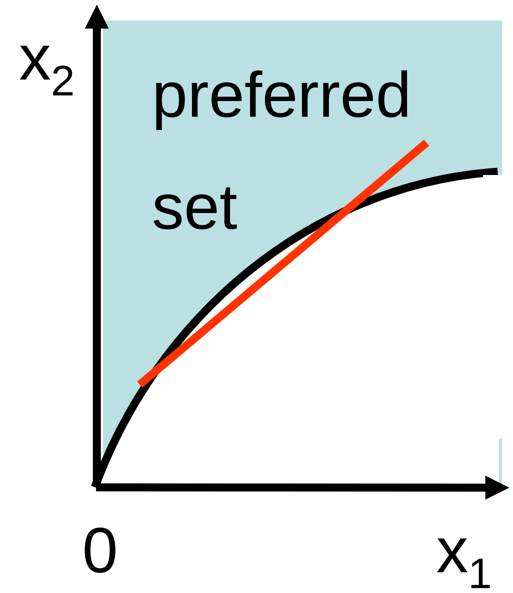
Figure 2



convexity yes

nonsatiation no

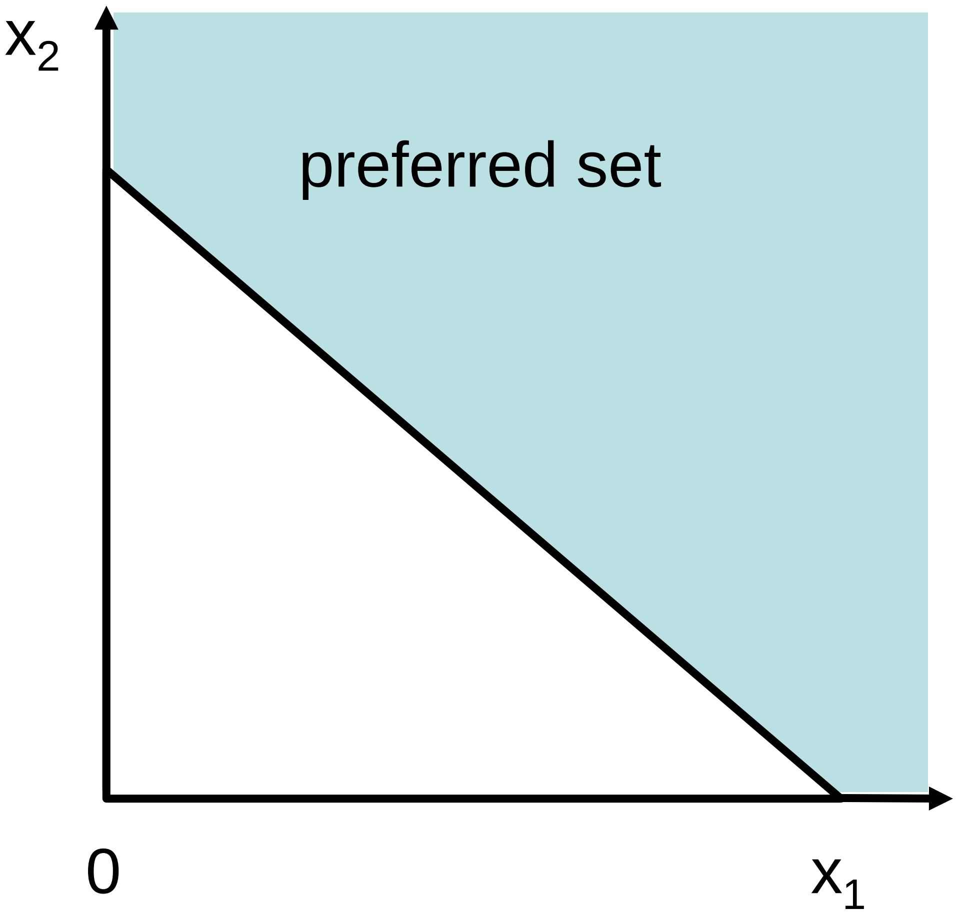
Figure 3



convexity no

nonsatiation no

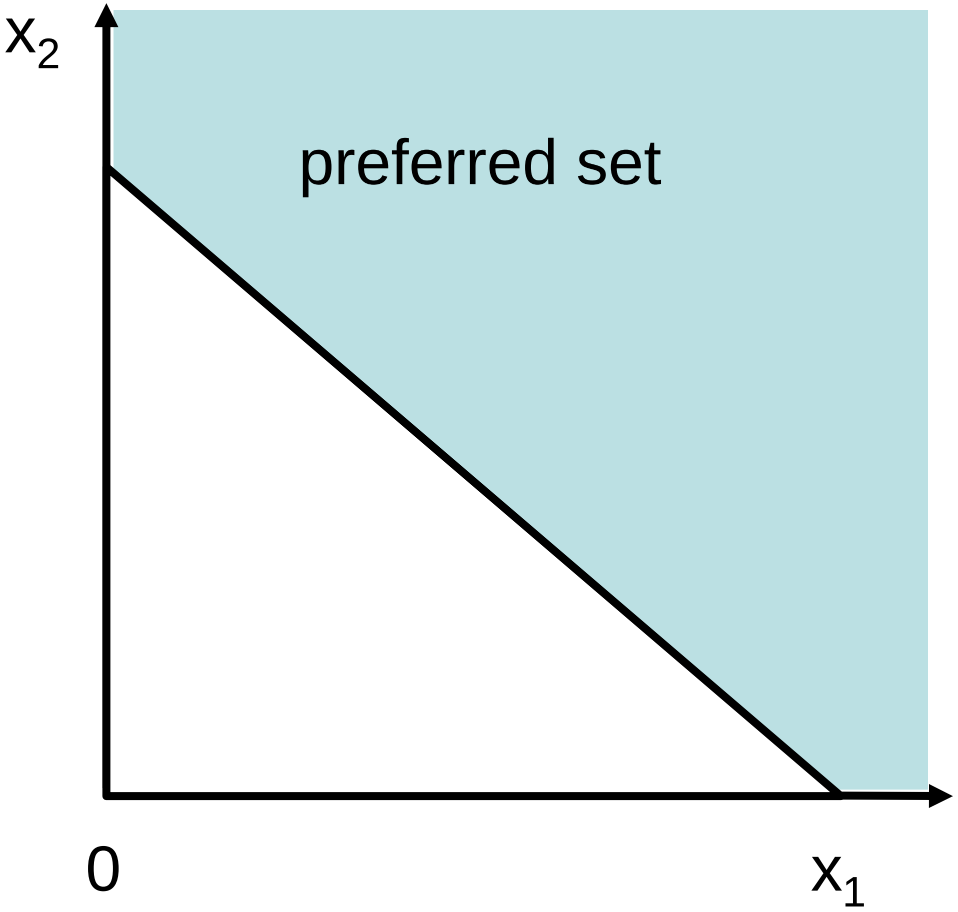
Do these preferences satisfy convexity and nonsatiation?



Convexity

Nonsatiation

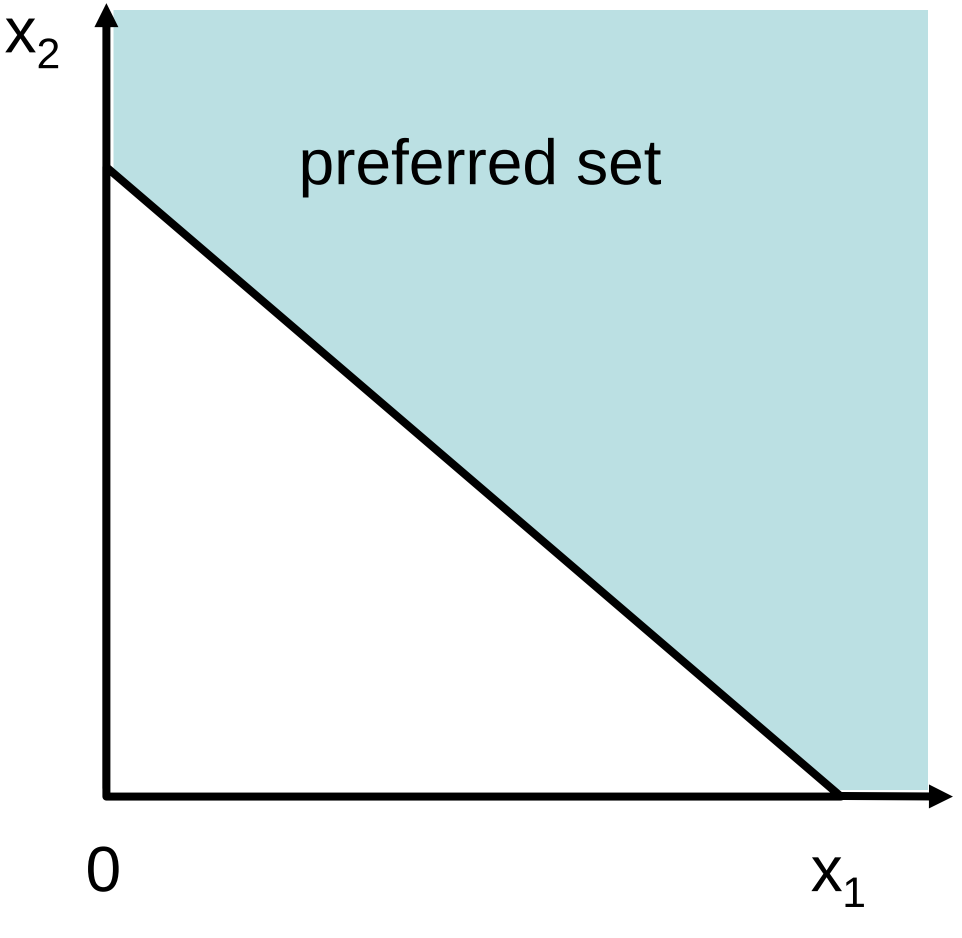
Do these preferences satisfy convexity and nonsatiation?



Convexity yes

Nonsatiation

Do these preferences satisfy convexity and nonsatiation?



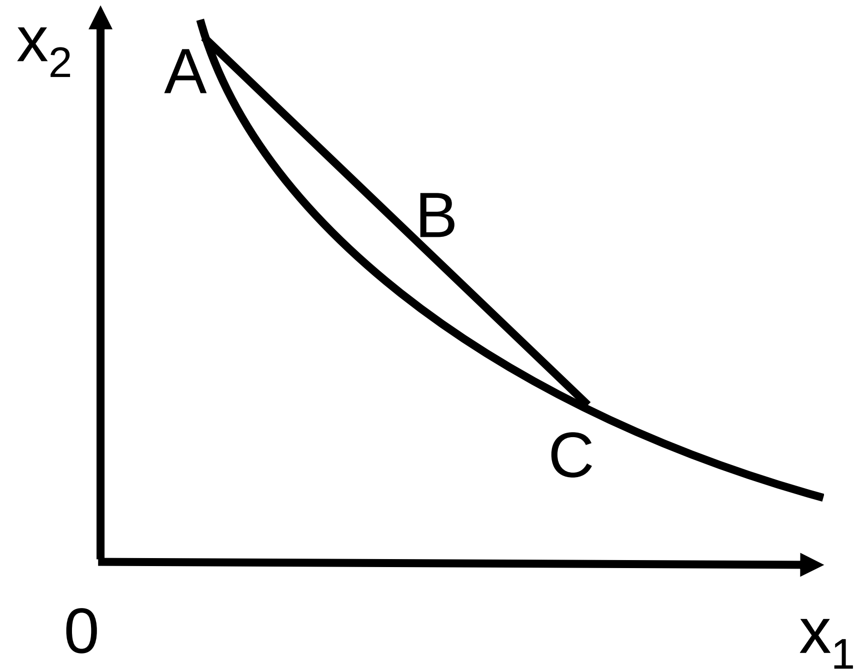
Convexity yes

Nonsatiation yes

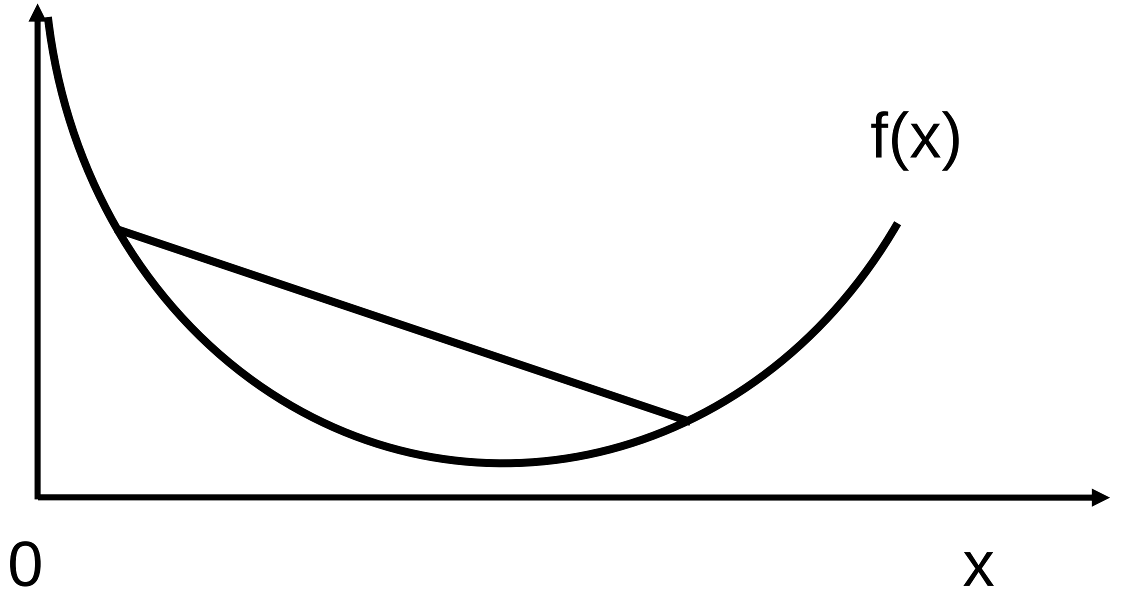
6. Checking for convexity

convex functions

A function is **convex** if any line joining two points on its graph lies entirely on or **above** the graph.

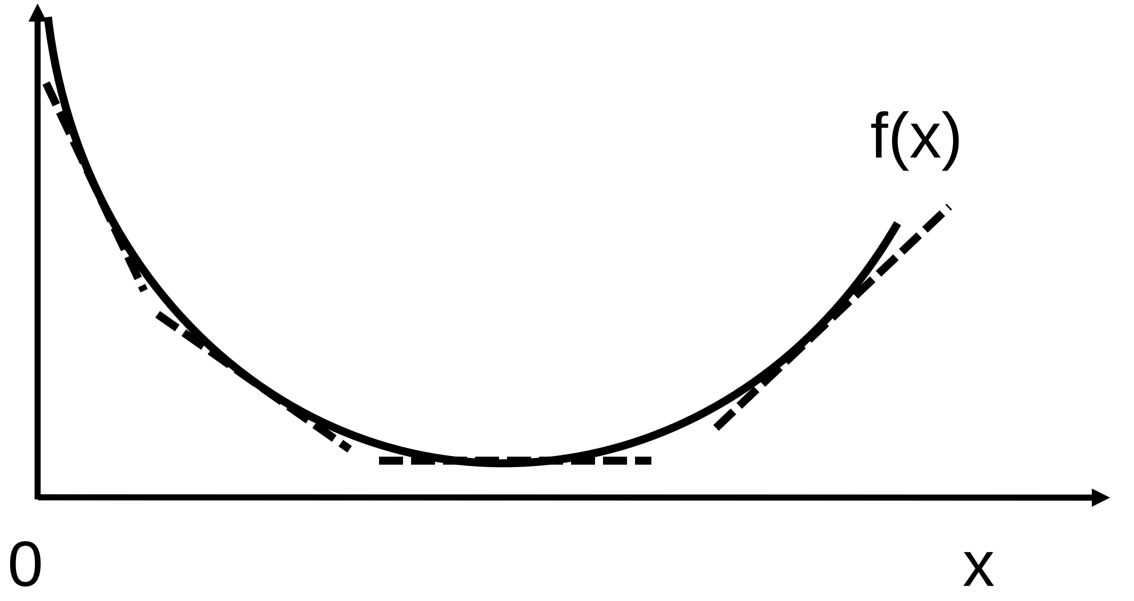


With 2 goods and nonsatiation the preferred **set is convex** if the indifference curve is a **convex function**.



Facts about convex functions

A function is convex if any line joining two points on its graph lies entirely on or below the graph.



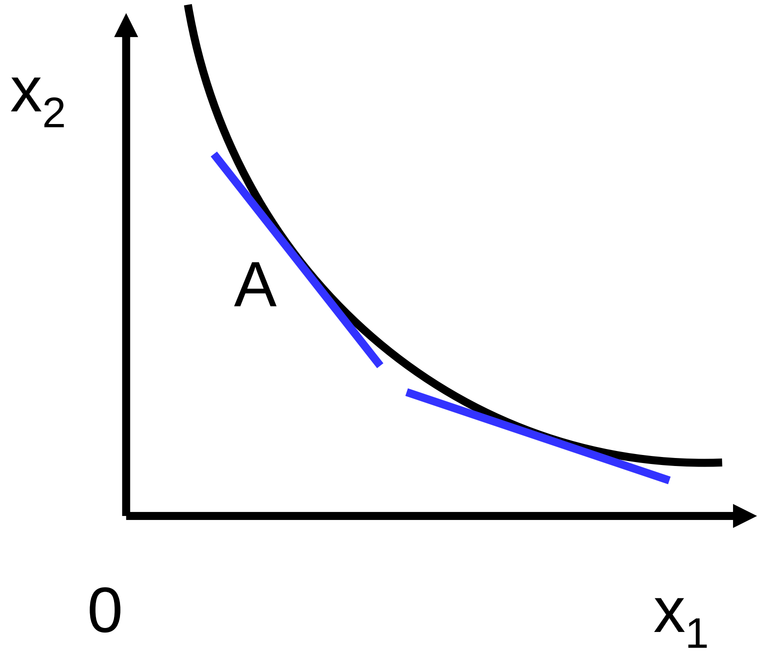
Facts about convex functions

For functions with first and second derivatives

Convex functions are functions with increasing first derivatives

Convex functions are functions with positive second derivatives.

Hicks assumed decreasing MRS



MRS = - gradient of indifference curve

Hicks assumes that the MRS decreases along an indifference curve as x_1 increases.

With 2 goods & nonsatiation decreasing MRS is the same as convexity.

How to check for convexity with two goods

- Use the formula for the utility function to x_2 as a function of x_1 & u.
- Find the second derivative of this function with respect to x_1.
- If the second derivative is positive the convexity assumption is satisfied.

- With more than two goods checking for convexity involves working with the second derivative matrix of utility. Not for EC201.

Example of checking for convexity

If $u = x_1^{2/5} x_2^{3/5}$ then

$x_2 = u^{5/3} x_1^{-2/3}$ so

$$\frac{\partial x_2}{\partial x_1} = -\ (2/3)\, u^{5/3} x_1^{-5/3}$$ and

$$\frac{\partial^2 x_2}{\partial x_1^2} = (10/9)\, u^{5/3} x_1^{-8/3} > 0$$

so the indifference curve is convex.

Assumptions of Consumer Theory

1. Completeness
2. Transitivity
3. Continuity
4. Nonsatiation
5. Convexity

We have now discussed all these.

We now look at some implications.

Now look at the implications of the assumptions on:

- Crossing of indifference curves
- Finding the Marginal Rate of Substitution (MRS)
- Using the MRS to see if two utility functions represent the same preferences
- Comparison of two people’s utility.

Can indifference curves cross?

7. Can indifference curves cross?

A and B are on the same

indifference curve so are

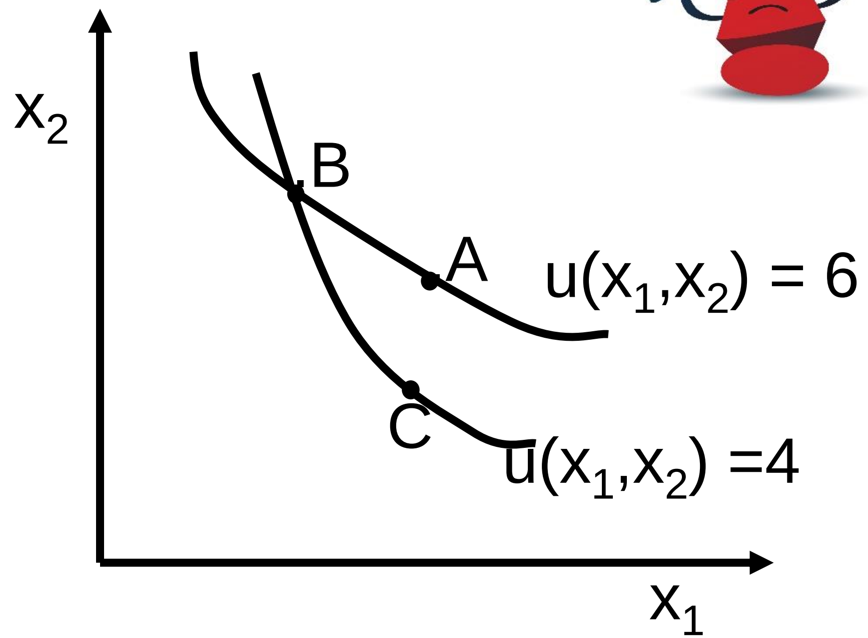
B and C are on the same

indifference curve so are

Transitivity implies that A and C

are.

But A has utility than C.

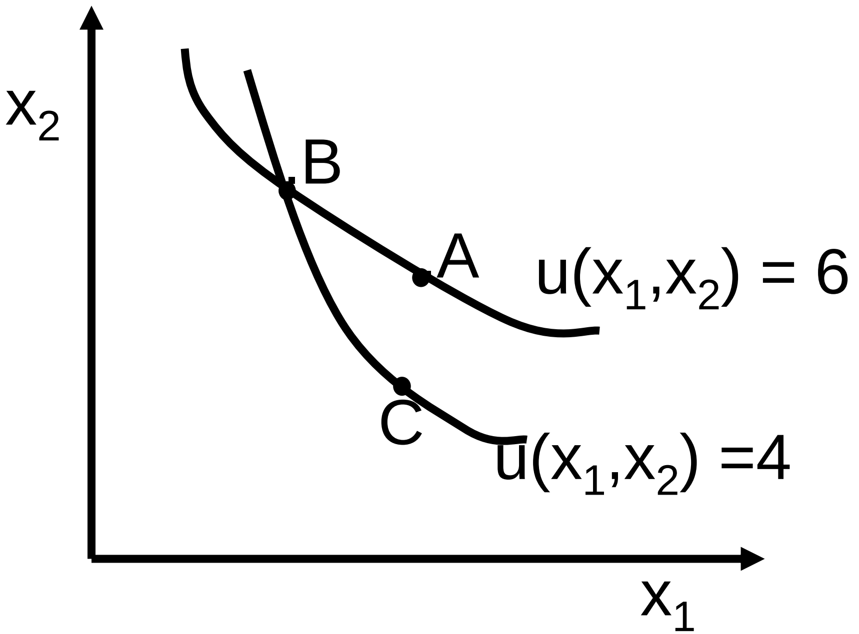


Transitivity implies indifference curves

Can indifference curves cross?

A and B are on the same indifference curve so are indifferent.

B and C are on the same indifference curve so are



Transitivity implies indifference curves

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A and B are on the same indifference curve so are indifferent.

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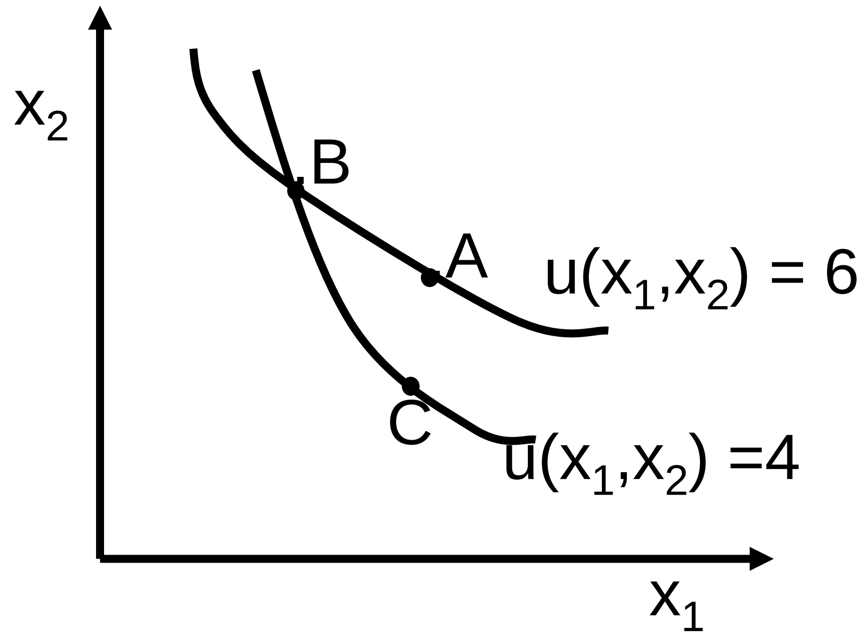


Transitivity implies indifference curves

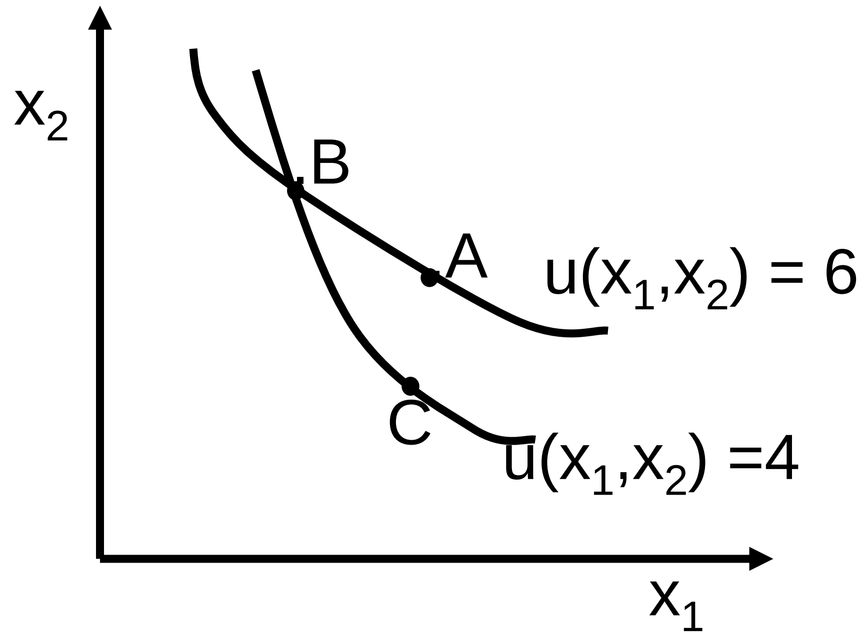
Can indifference curves cross?

A and B are on the same indifference curve so are indifferent.

B and C are on the same indifference curve so are indifferent.

Transitivity implies that A and C are indifferent.

But A has utility than C.



Transitivity implies indifference curves

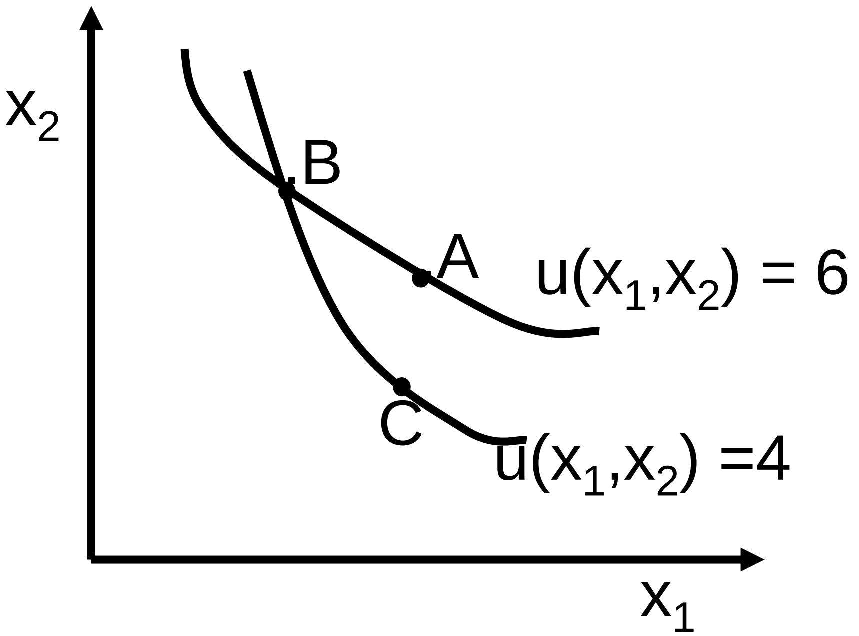
Can indifference curves cross?

A and B are on the same indifference curve so are indifferent.

B and C are on the same indifference curve so are indifferent.

Transitivity implies that A and C are indifferent.

But A has higher utility than C.



Transitivity implies indifference curves

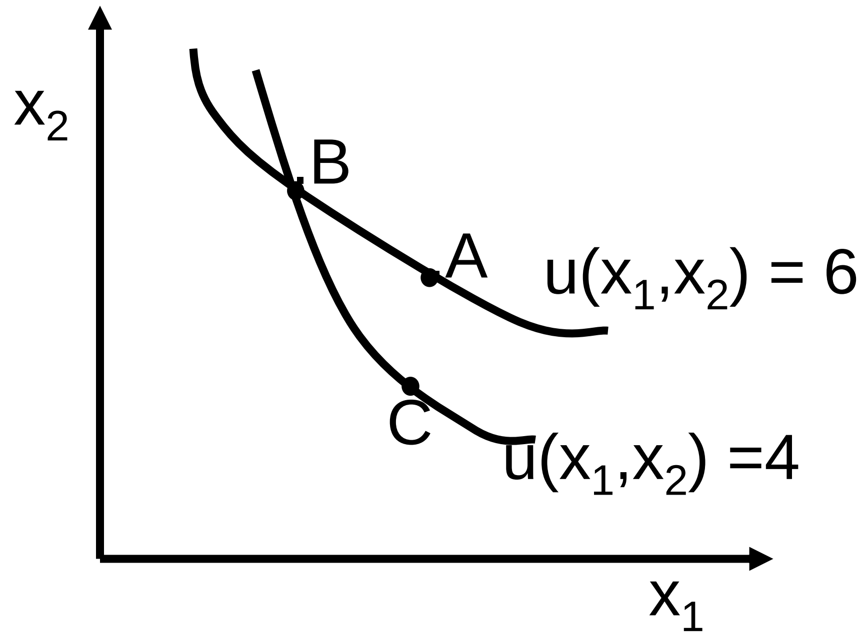
Can indifference curves cross?

A and B are on the same indifference curve so are indifferent.

B and C are on the same indifference curve so are indifferent.

Transitivity implies that A and C are indifferent.

But A has higher utility than C.



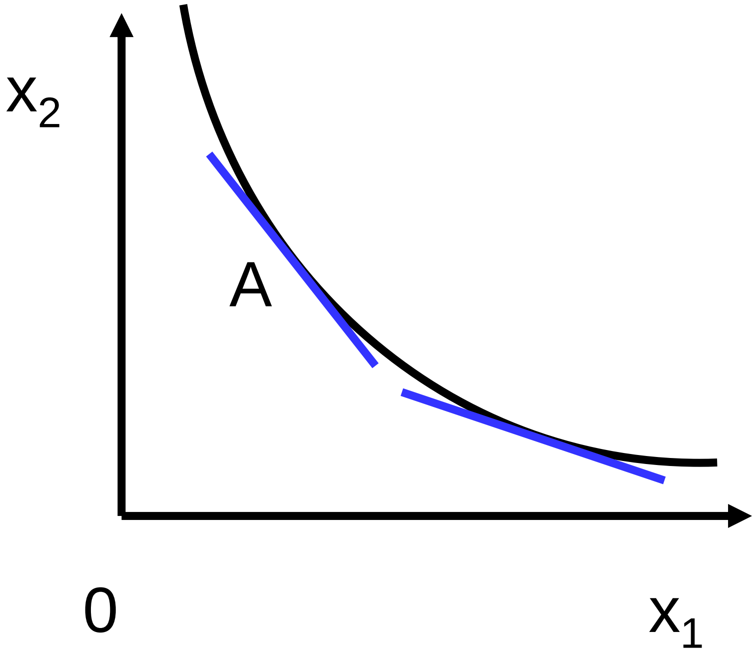
Transitivity implies indifference curves don't cross.

Marginal Rate of Substitution (MRS)

8. Finding the marginal rate of substitution

- Definition and formula
- Explaining the formula
- Example: Cobb-Douglas utility function
- Using the MRS to find whether two utility functions represent the same preferences
- Convexity and MRS

Marginal Rate of Substitution: Definition

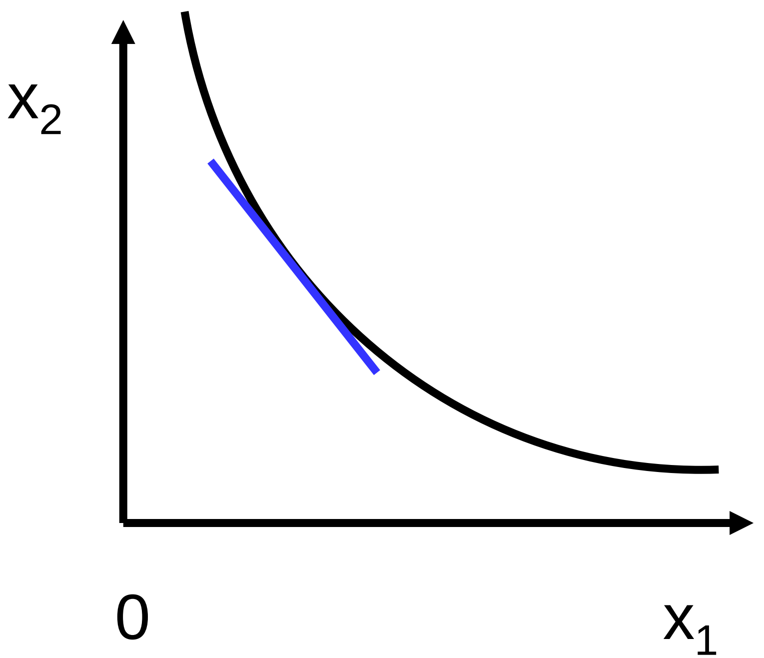


Hicks and most textbooks defines the MRS as
 - gradient of indifference curve.

Perloff defines the MRS as
 - gradient of indifference curve

Marginal Rate of Substitution: Formula

$$\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}}$$



Explaining the formula for MRS

If x_2 stays at x_{2A}

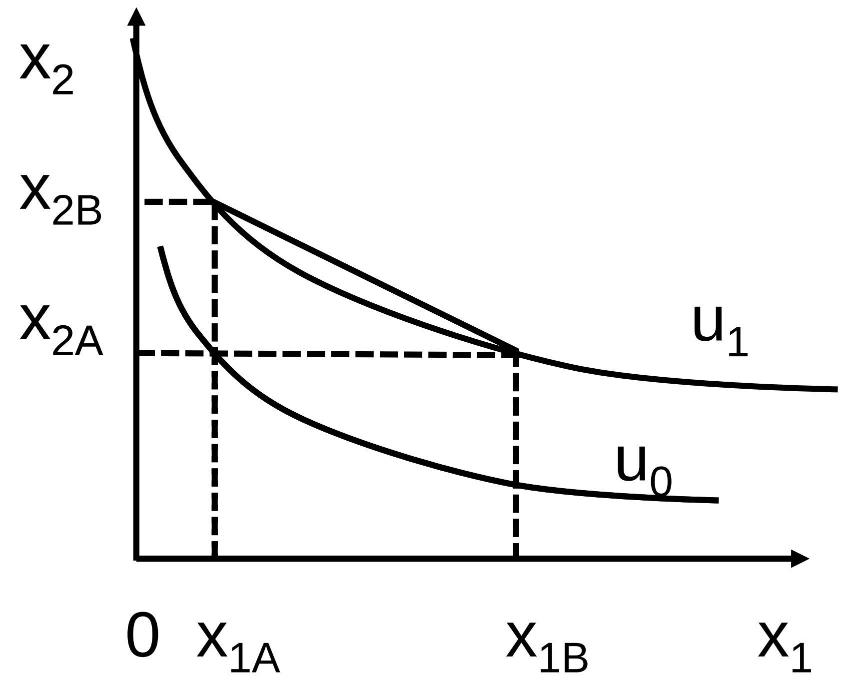
x_1 changes from x_{1A} to x_{1B}

u changes from u_0 to u_1

Then

$$\frac{(u_1 - u_0)}{(x_{1B} - x_{1A})} \approx \frac{\partial u}{\partial x_1}$$

so $$u_1 - u_0 \approx (x_{1B} - x_{1A}) \frac{\partial u}{\partial x_1}$$



Explaining the formula for MRS

If x_1 stays at x_{1A}

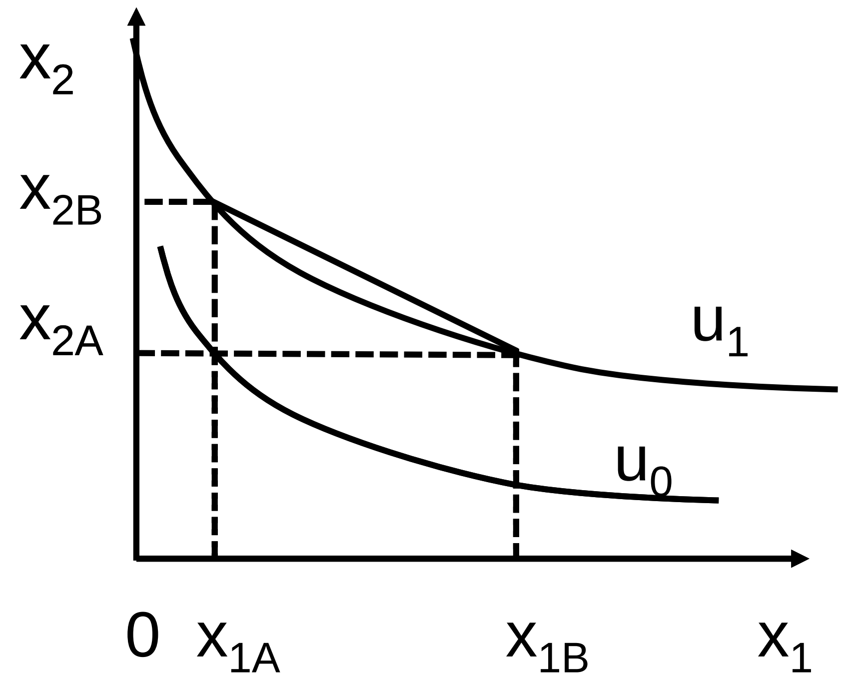
x_2 changes from x_{2A} to x_{2B}

u changes from u_0 to u_1

Then

$$\frac{(u_1 - u_0)}{(x_{2B} - x_{2A})} \approx \frac{\partial u}{\partial x_2}$$

so $$u_1 - u_0 \approx (x_{2B} - x_{2A}) \frac{\partial u}{\partial x_2}$$



Explaining the formula for MRS

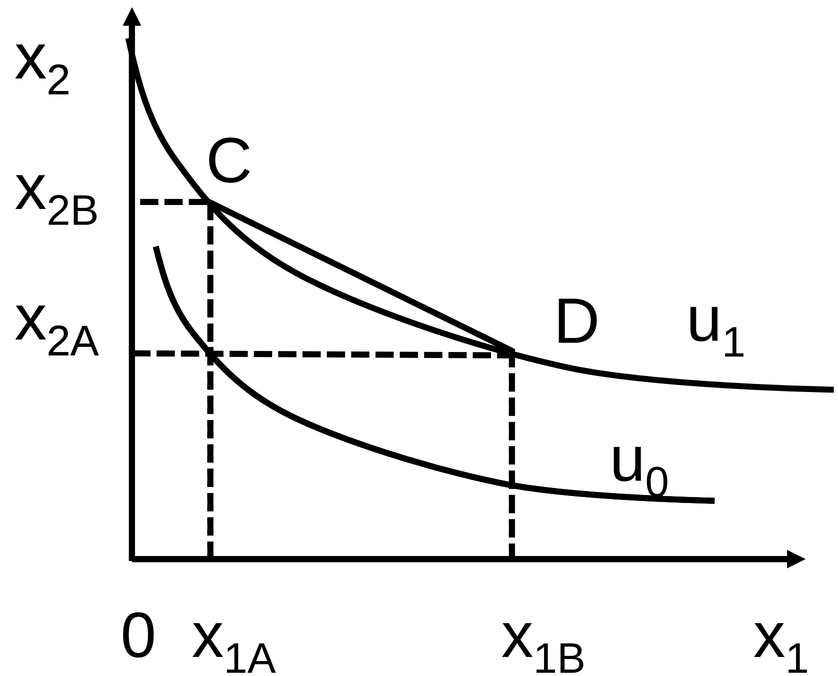
From the previous slides

$$(u_1 - u_0) \approx (x_{1B} - x_{1A}) \frac{\partial u}{\partial x_1}$$

$$\approx (x_{2B} - x_{2A}) \frac{\partial u}{\partial x_2}$$

So slope of line CD =

$$-\frac{(x_{2B} - x_{2A})}{(x_{1B} - x_{1A})} \approx -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}}$$

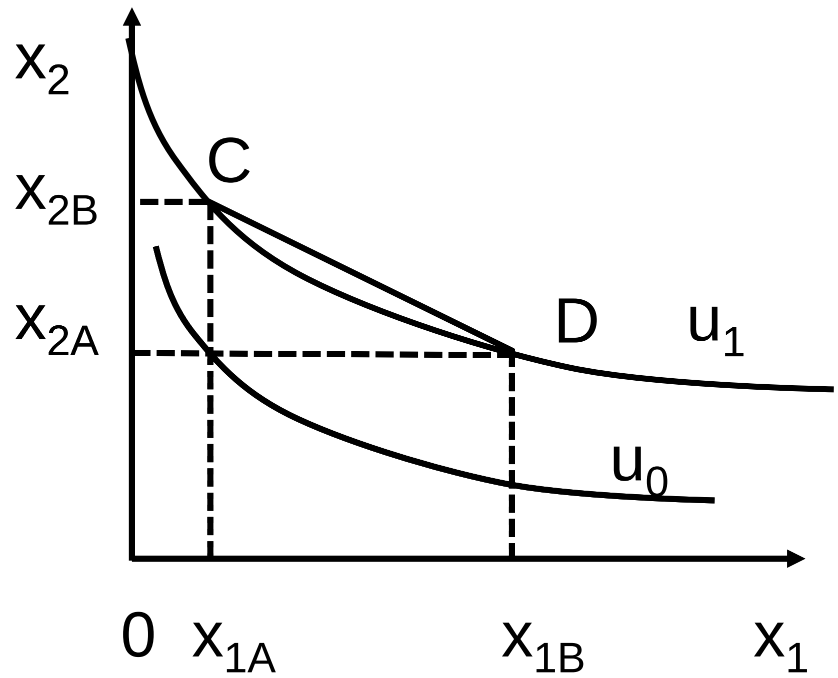


Explaining the formula for MRS

When C and D are close the line CD is close to a tangent

Gradient of the indifference curve = - MRS

$$MRS = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}}$$



MRS example: Cobb-Douglas utility function

utility $u(x_1,x_2) = u = x_1^{2/5}x_2^{3/5}$

MRS

$$-\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{\frac{2}{5}x_1^{-3/5}x_2^{3/5}}{\frac{3}{5}x_1^{2/5}x_2^{-2/5}} = -\frac{2x_2}{3x_1}$$

Using the MRS to find whether two utility functions represent the same preferences

If f is a strictly increasing function and $\hat{u}(x_1,x_2) = f(u(x_1,x_2))$ then u and û represent the same preferences.

What if you have two utility functions and cannot see whether they represent the same preferences?

Same preferences imply same indifference curves imply same MRS.

Check whether utility functions they have the same MRS to see whether they represent the same preferences..

Using the MRS to find whether two utility functions represent the same preferences

MRS from utility function $u(x_1,x_2)$

$$\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}}$$

Using the chain rule MRS from utility function

$\hat{u}(x_1,x_2) = f(u(x_1,x_2))$

$$\frac{\frac{\partial \hat{u}}{\partial x_1}}{\frac{\partial \hat{u}}{\partial x_2}} = \frac{\frac{df}{du}\frac{\partial u}{\partial x_1}}{\frac{df}{du}\frac{\partial u}{\partial x_2}} = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}}$$

Questions on preferences and utility

Can you compare different people's utility?

- Consumer theory does not give a way of doing this.

Can you compare different people’s utility?

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- Most policy changes are not Pareto improving.

Can you compare different people's utility?

- Consumer theory does not give a way of doing this.
- Most policy changes are not Pareto improving.
- Ethical and political value judgements matter.

Regret

Have you ever chosen to do or consume something and then regretted it?

Choice under uncertainty?

Inability to commit yourself?

How far does our welfare depend on

- relationships with other people?

How far does our welfare depend on

- relationships with other people?
- What we think of ourselves?

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- What we think of ourselves?
- What other people think of us?

How far does our welfare depend on

- relationships with other people?
- What we think of ourselves?
- What other people think of us?
- What we think other people think of us?

Questions on preferences

- Are people rational?
- Who is the consumer?
- What is the effect of other consumers on your choices?
- What is the relationship between utility, welfare and happiness?
- What is the effect of what we buy and sell on our quality of life?
- Should we take psychology seriously?
- Is the simple model useful?

Where we have got to?

- We have thought critically about the assumptions (completeness, transitivity, continuity, nonsatiation & convexity)
- we have got ordinal utility & indifference curves from the assumptions
- We can use calculus to
  - check for nonsatiation & convexity
  - find the MRS
  - see if two different utility functions represent the same preferences.